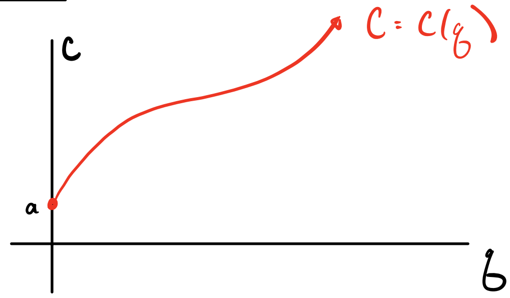
§3.2 Introduction to Marginal Analysis

Let $C(q)$ = total cost of producing q units

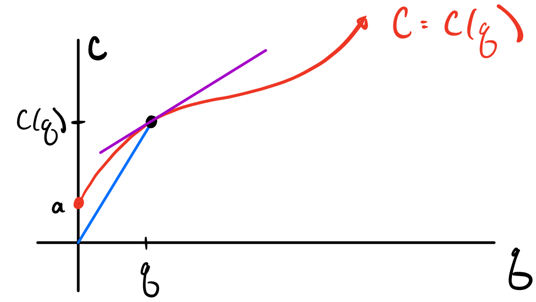
e.g. $C(q) = \underset{\uparrow \text{ Fixed Costs}}{a} + \underbrace{bq + cq^2 + dq^3}_{\text{Variable Costs}}$



When producing q units,

- Average Cost $= \dfrac{C(q)}{q}$ \$/unit
- Marginal Cost $= C'(q) = \dfrac{dC}{dq}$ \$/unit

(Marginal = Derivative)



EXAMPLE 1 Average Cost and Marginal Cost

Suppose a company has estimated that the cost, in dollars, of producing q items per week is $C(q) = 3000 + 13q - 0.01q^2 + 0.000003q^3$.

(a) What are the fixed costs?

(b) Find a function for the average cost of each unit produced. What is the average cost when 1500 items are produced?

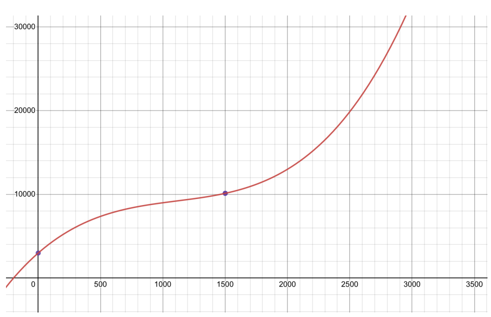
(c) Find the marginal cost function. What is the marginal cost when 1500 units are produced?

(d) What is the actual cost of the 1501st item?

$\dfrac{C(1500)}{1500} = \dfrac{\$10{,}125}{1500 \text{ units}} = \$6.75/\text{unit}$

$C'(1500) = \$3.25/\text{unit}$

$C(1501) - C(1500) \approx \$10{,}128.2535 - \$10{,}125 \approx \$3.2535$

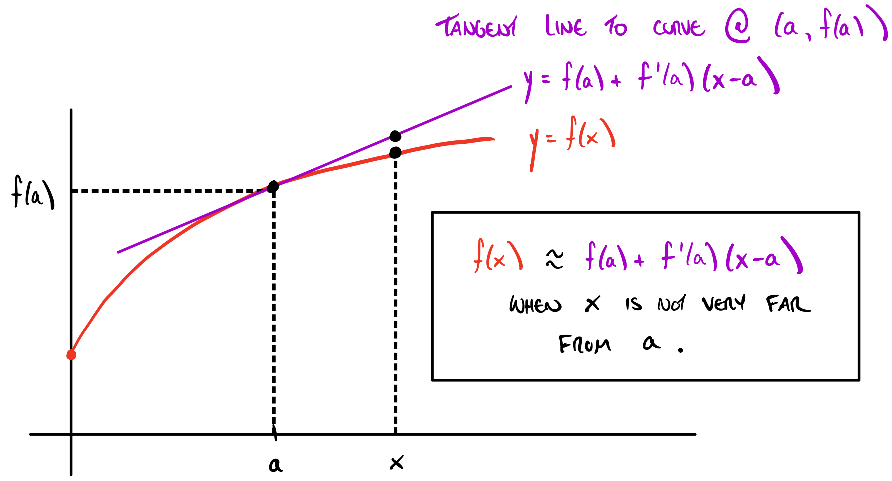


q	$3000 + 13q - .01q^2 + .000003q^3$
0	3000
1500	10125
1501	10128.254

Note: Marginal Cost $C'(q) \approx$ cost of $(q+1)^{st}$ unit

$$C'(q) = \lim_{h \to 0} \frac{C(q+h) - C(q)}{h} \underset{(h=1)}{\approx} C(q+1) - C(q) \checkmark$$

Linear Approximation



$$f(x) \approx f(a) + f'(a)(x-a)$$

when x is not very far from a.

We can use the values $C(q_0)$ & $C'(q_0)$ to estimate the cost of producing q units when q is close to q_0.

$$\boxed{C(q) \approx C(q_0) + C'(q_0)(q-q_0)}$$

Let $q - q_0 = \Delta q$ $\quad (q = q_0 + \Delta q)$

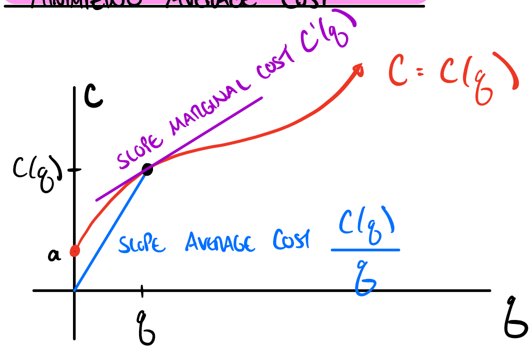
$$\boxed{C(q_0 + \Delta q) \approx C(q_0) + \underbrace{C'(q_0)\Delta q}}$$

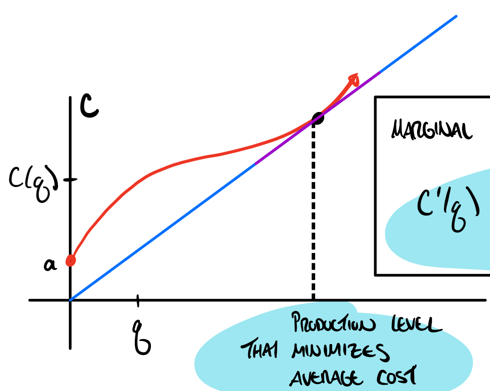
$\approx$ cost of producing an additional Δq units once q_0 units are already being produced.

12. Water purification If $C(v)$ is the cost, in dollars, of purifying v gallons of drinking water, and $C'(200{,}000) = 0.26$, estimate the cost of purifying an additional 3000 gallons of water once 200,000 gallons have been processed.

$$C'(q_0)\Delta q = (.26)(3000) = \$780$$

Minimizing Average Cost



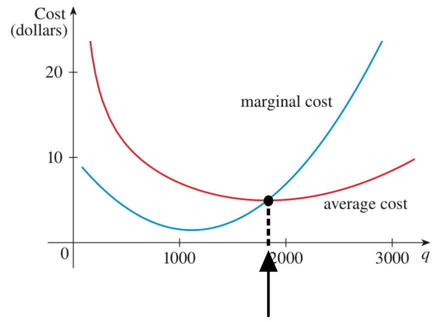


Marginal cost = ave. cost.

$$C'(q) = \frac{C(q)}{q}$$

$C'(q) < \dfrac{C(q)}{q}$ MARGINAL COST < AVE. COST $\Rightarrow$ INCREASING PRODUCTION WILL LOWER AVERAGE COST

$C'(q) > \dfrac{C(q)}{q}$ MARGINAL COST > AVE. COST $\Rightarrow$ INCREASING PRODUCTION WILL RAISE AVERAGE COST



PRODUCTION LEVEL THAT MINIMIZES AVERAGE COST.

16. Appliance production A manufacturer's weekly cost, in dollars, for producing q lamps is

$$C(q) = 810 + 3q + 0.002q^2$$

Find the number of lamps that should be produced in order to minimize the average cost.

$$\frac{810 + 3q + .002q^2}{q} = 3 + .004q$$

$$810 + \cancel{3q} + .002q^2 = \cancel{3q} + .004q^2$$

$$810 = .002q^2$$

$$405{,}000 = q^2 \Rightarrow q \approx 636$$

REVENUE & PROFIT

LET $R(q)$ = TOTAL REVENUE FOR PRODUCING q UNITS

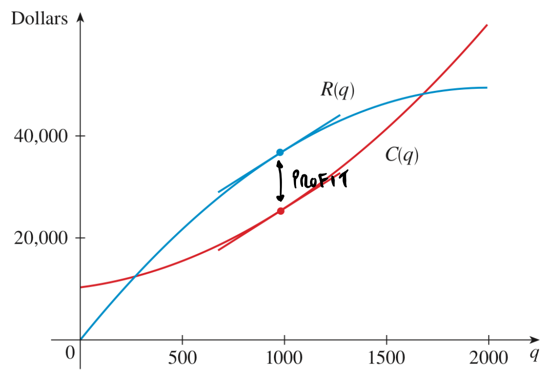
- $\dfrac{R(q)}{q}$ = AVERAGE REVENUE PER UNIT
- $R'(q)$ = MARGINAL REVENUE $\approx$ REVENUE GAINED BY PRODUCING 1 ADDITIONAL UNIT.
- $P(q) = R(q) - C(q)$ = PROFIT

$R'(q) > C'(q)$ MARGINAL REVENUE > MARGINAL COST $\Rightarrow$ INCREASING PRODUCTION WILL INCREASE PROFITS

$R'(q) < C'(q)$ MARGINAL REVENUE < MARGINAL COST $\Rightarrow$ INCREASING PRODUCTION WILL LOWER PROFITS

$\therefore$ $\boxed{\text{PROFIT } P(q) \text{ IS MAXIMIZED WHEN } R'(q) = C'(q)}$

to maximize profit, production should be increased to the point at which marginal revenue and marginal cost are equal



20. Food production A food maker estimates that the monthly cost, in dollars, for producing b boxes of its breakfast cereal is

$$C(b) = 1200 + 0.9b + 0.0002b^2$$

The revenue, in dollars, the company earns from selling b boxes of the cereal is given by

$$R(b) = 2.8b - 0.0001b^2$$

(a) Write equations for the average revenue and marginal revenue functions.

(b) Find the number of boxes of cereal that the company should produce monthly in order to maximize profit.

$C'(q) = R'(q)$

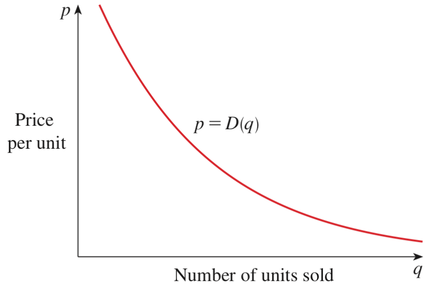
$.9 + .0004b = 2.8 - .0002b$

$.0006b = 1.9$

$b \approx$ 3167 BOXES.

DEMAND

LET $p = D(q)$ BE THE PRICE PER UNIT THAT A COMPANY CAN CHARGE IF IT SELLS q UNITS.
(D IS CALLED PRICE / DEMAND FUNCTION, GRAPH IS CALLED DEMAND CURVE)



NOTE: $\boxed{R(q) = q \cdot D(q)}$

■ EXAMPLE 5 A Demand Function and Maximizing Profit

A company has cost and demand functions

$$C(q) = 84 + 1.26q - 0.01q^2 + 0.00007q^3 \qquad \text{and} \qquad D(q) = 3.5 - 0.01q$$

(a) If the price of each unit is \$1.20, how many units will be sold?

(b) Determine the production level that will maximize profit for the company.

(a) $1.20 = 3.5 - .01q \Rightarrow q = 230$

(b) $C'(q) = R'(q)$: $1.26 - \cancel{.02q} + .00021q^2 = 3.5 - \cancel{.02q}$

$$q = \sqrt{\frac{2.24}{.00021}} \approx 103 \text{ UNITS}$$