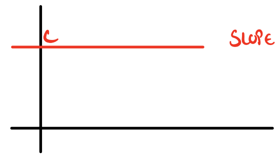
§3.1 SHORTCUTS TO FINDING DERIVATIVES

DERIVATIVE OF A CONSTANT $f(x) = c$:

$$\boxed{\frac{d}{dx}[c] = 0}$$



SLOPE 0.

PROOF: $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to h} \dfrac{c - c}{h} = 0$ ✓

DERIVATIVE OF A POWER FUNCTION $f(x) = x^n$, $n = 1, 2, 3, \ldots$

THE POWER RULE:

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$$

THIS ACTUALLY WORKS FOR ANY EXPONENT n

NOTE:

$$(x+h)^2 = x^2 + 2xh + h^2$$
$$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$
$$(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

$$(x+h)^n = x^n + nx^{n-1}h + h^2(\ldots)$$

PROOF: $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} \dfrac{(x+h)^n - x^n}{h}$

$$= \lim_{h\to 0} \frac{x^n + nx^{n-1}h + h^2(\ldots) - x^n}{h} = \lim_{h\to 0} \frac{\cancel{h}\left(nx^{n-1} + h(\ldots)\right)}{\cancel{h}} = nx^{n-1} \checkmark$$

EX. IF $f(x) = x^6$ AND $g(x) = \dfrac{1}{x\sqrt{x}}$, FIND $f'(x)$ & $g'(x)$.

THE CONSTANT MULTIPLE RULE:

$$\frac{d}{dx}\left[cf(x)\right] = c\frac{d}{dx}\left[f(x)\right]$$
$$= cf'(x)$$

PROOF: $\lim\limits_{h\to 0} \dfrac{cf(x+h) - cf(x)}{h} = \left(\lim\limits_{h\to 0} c\right)\left(\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}\right)$

$= cf'(x)$ ✓

THE SUM/DIFFERENCE RULE:

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}\left[f(x)\right] \pm \frac{d}{dx}\left[g(x)\right]$$
$$= f'(x) \pm g'(x)$$

PROOF: $$\lim_{h\to 0} \frac{(f(x+h) \pm g(x+h)) - (f(x) \pm g(x))}{h}$$

$$= \lim_{h\to 0} \frac{(f(x+h) - f(x)) \pm (g(x+h) - g(x))}{h}$$

$$= \lim_{h\to 0} \left[\frac{f(x+h) - f(x)}{h} \pm \frac{g(x+h) - g(x)}{h} \right]$$

$$= \lim_{h\to 0} \left[\frac{f(x+h) - f(x)}{h} \right] \pm \lim_{h\to 0} \left[\frac{g(x+h) - g(x)}{h} \right] = f'(x) \pm g'(x) \quad \checkmark$$

ex. LET $f(x) = 8x^4 - 5x^3 + 6^2 - \frac{4}{x^3}$. FIND $f'(x)$.

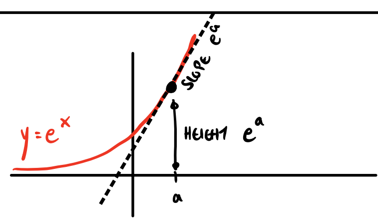
ex. LET $g(t) = \sqrt[4]{x} + \frac{2}{\sqrt[3]{x}} - 5x^{6/13}$. FIND $g'(t)$.

EXAMPLE 8

Identifying Points on a Curve with Horizontal Tangent Lines

Find the points on the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal.

$$\boxed{\frac{d}{dx}\left[e^x\right] = e^x}$$



PROOF: $$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h\to 0} \frac{e^x(e^h - 1)}{h}$$

$$= e^x \cdot \lim_{h\to 0} \frac{e^h - e^0}{h} = e^x \cdot \underbrace{f'(0)}_{\text{SLOPE OF } e^x \text{ AT } x=0 \text{ IS } 1} = e^x$$

(DEF. OF e)

ex. LET $f(x) = 2e^x - 5x + 3$.
FIND POINT OF CURVE $y = f(x)$ WHERE TANGENT LINE IS HORIZONTAL.

https://www.desmos.com/calculator/6pjvnjluje

ex. $\frac{d}{dx}\left[\sqrt{x}(2x+3)(3x-5)\right]$

DISTRIBUTE FIRST !

ex. $\frac{d}{dx}\left[\frac{(x+1)^3}{x^2}\right]$

ex. GIVE AN EQ OF TANGENT LINE TO $y=(1+2x)^2$ AT $(1,9)$

$$y=f(a)+f'(a)(x-a)$$

64. Motion The equation of motion of a particle is $s = 2t^3 - 7t^2 + 4t + 1$, where s is in meters and t is in seconds.

(a) Find the velocity and acceleration as functions of t.

(b) Find the acceleration after 1 second.