§2.2 LIMITS

CONSIDER $f(x) = \frac{x-1}{x^2-1}$. DOM$(f) = (-\infty,-1) \cup (-1,1) \cup (1,\infty)$

$f(1)$ = UNDEFINED BUT

x	$f(x)$	x	$f(x)$
0.8	0.55556	1.2	0.45455
0.9	0.52632	1.1	0.47619
0.95	0.51282	1.05	0.48780
0.98	0.50505	1.02	0.49505
0.99	0.50251	1.01	0.49751
0.995	0.50125	1.005	0.49875
0.999	0.50025	1.001	0.49975
$\downarrow 1^-$	$\downarrow \frac{1}{2}$	$\downarrow 1^+$	$\downarrow \frac{1}{2}$

AS THE INPUT x GETS CLOSER TO 1, THE OUTPUT $f(x)$ GETS CLOSER TO $\frac{1}{2}$.

THE LIMIT OF $f(x)$ AS x APPROACHES 1 IS $\frac{1}{2}$

$$\lim_{x\to 1} \frac{x-1}{x^2-1} = \frac{1}{2}$$

Definition We write

$$\lim_{x\to a} f(x) = L$$

and say "the limit of $f(x)$, as x approaches a, equals L"

if the values of $f(x)$ approach L as the values of x approach a (but are not equal to a).

NOTE: ONE WAY TO APPROXIMATE A LIMIT $\lim_{x\to a} f(x)$ IS TO PLUG IN NUMBERS THAT GET CLOSER & CLOSER TO a & TRY TO FIGURE OUT WHAT # (IF ANY) THE OUTPUTS GET CLOSER & CLOSER TO.

Limit Laws Suppose that c is a constant and the limits

$$\lim_{x\to a} f(x) = A \quad \text{and} \quad \lim_{x\to a} g(x) = B$$

exist. Then

1. $\lim_{x\to a} [f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$ $= A + B$
2. $\lim_{x\to a} [f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$ $= A - B$
3. $\lim_{x\to a} [cf(x)] = c \lim_{x\to a} f(x)$ $= cA$
4. $\lim_{x\to a} [f(x)g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$ $= AB$
5. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$ $= \frac{A}{B}$
6. $\lim_{x\to a} [f(x)]^n = [\lim_{x\to a} f(x)]^n$ where n is a positive integer $= A^n$

FACTS (1) $\lim_{x\to a} c = c$

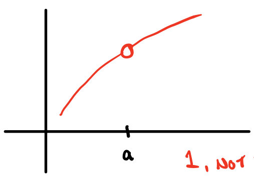
(2) $\lim_{x\to a} x = a$

ex. Evaluate $\lim\limits_{x \to 2} \dfrac{2x^3 - 1}{5x^2 + 4x}$ (Can evaluate by direct substitution)

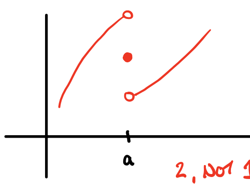
Definition A function f is **continuous at a number a** if

$$\lim_{x \to a} f(x) = f(a)$$

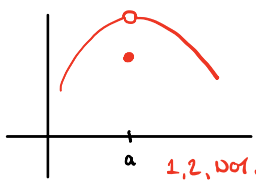
1. Limit exists
2. $f(a)$ is defined
3. The two are equal



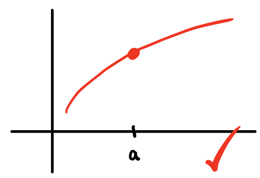
1, not 2



2, not 1



1, 2, not 3



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Def: A function is **continuous on an interval** if it is continuous at every point in that interval.

That is, its graph can be drawn without lifting pencil off paper.

THM.

The following types of functions are continuous at every number in their domains:

linear functions, polynomials, rational functions

power functions, root functions

exponential functions, logarithmic functions

20. (a) Find the domain of $g(t) = \dfrac{t^2 - 3t - 4}{t + 1}$.

(b) Find $\lim\limits_{t \to 3} g(t)$.

(c) Find $\lim\limits_{t \to -1} g(t)$.

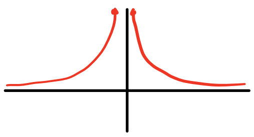
Theorem: If $f(x) = g(x)$ for all x on an interval containing a, except possibly at a, then

$$\lim_{x \to a} f(x) = \lim_{x \to a} g(x).$$

35. $\lim\limits_{x \to 7} \dfrac{\sqrt{x + 2} - 3}{x - 7}$ (Rationalize numerator)

38. $\lim\limits_{t \to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2 + t} \right)$

INFINITE LIMITS



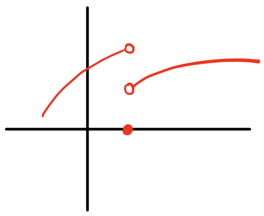
$$\lim_{x \to 0} \frac{1}{x^2} = \infty$$
$$= \text{D.N.E.}$$

Both correct.

ONE SIDED LIMITS

$\lim_{x \to a^-} f(x) = L$ / $\lim_{x \to a^+} f(x) = L$: AS x GETS CLOSER TO a, ALWAYS $x < a$ / $x > a$, $f(x)$ GETS CLOSER TO L.

ex.



$\lim_{x \to 1^-} f(x) = 2$ $\quad \lim_{x \to 1^+} f(x) = 1$ $\quad (f(1) = 0)$

NOTE: $\lim_{x \to a} f(x) = L$ IF & ONLY IF $\lim_{x \to a^-} f(x) = \lim_{x \to a^+}$

ex. Let
$$f(x) = \begin{cases} x+3 & \text{if } x \le -2 \\ x^2 & \text{if } -2 < x < 1 \\ 4 & \text{if } x = 1 \\ \sqrt{x} & \text{if } x > 1 \end{cases}$$

(a) $\lim_{x \to -2^-} f(x)$

(b) $\lim_{x \to -2^+} f(x)$

(c) $\lim_{x \to 1^-} f(x)$

(d) $\lim_{x \to 1^+} f(x)$

50. Let $F(x) = \dfrac{x^2 - 1}{|x - 1|}$.

(a) Find

(i) $\lim_{x \to 1^+} F(x)$ **(ii)** $\lim_{x \to 1^-} F(x)$

(b) Does $\lim_{x \to 1} F(x)$ exist?

(c) Sketch the graph of F.