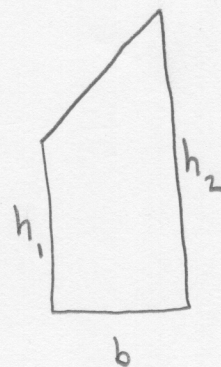


§ 14. FUNCTIONS OF SEVERAL VARIABLES

Area of a rectangle $A(l, w) = lw$

Volume of a box $V(l, w, h) = lwh$

Area of a trapezoid $A(b, h_1, h_2) = \frac{1}{2} b(h_1 + h_2)$



Distance from a point to $(3, 4, 5)$

$$d(x, y, z) = \sqrt{(x-3)^2 + (y-4)^2 + (z-5)^2}$$

Def: A function of 2 (3) variables is a rule that assigns to each ordered pair (triple) of real numbers (x, y) (x, y, z) in a set D a unique real number $f(x, y)$ $(f(x, y, z))$.

The set D is the domain and the set of values f takes on is the range.

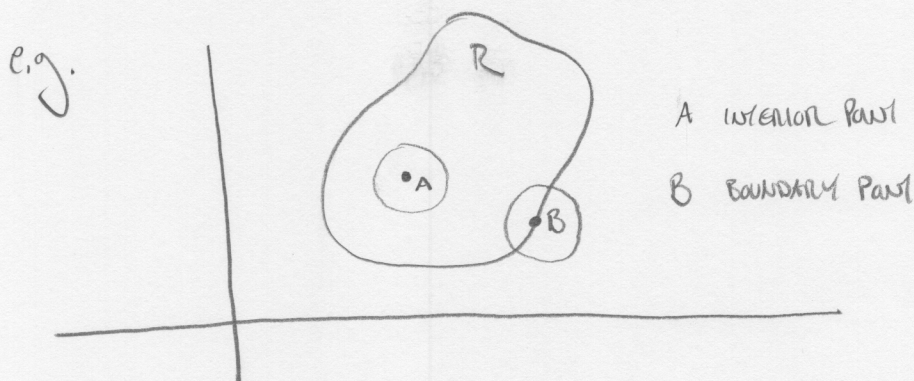
ex. $f(x, y) = \sqrt{xy}$, $f(4, 9)$, sketch domain

$f(x, y) = \frac{\sqrt{y-3x+5}}{x^2-4}$, $f(1, 2)$, sketch domain

$f(x, y) = \sin^{-1}(x^2 + y^2 - 2)$, $f(1, 1)$, sketch domain

Def. A subset of the xy -plane is called a region R ,
(The domain of a func. of 2 var. is a region)

A point (x_0, y_0) is called an interior point of R
if it is the center of a disk (maybe very small radius)
contained in R . It is called a boundary point if
every disk centered at this point contains points not inside R .



A region R is called open if every point in R
is an interior point, and closed if it contains
all of its boundary points.

A region R is called bounded if it is contained
in some disk of finite radius. Otherwise it is
called unbounded.

ex. FIND DOMAIN & FIND BOUNDARY POINTS OF DOMAIN.

IS THE DOMAIN OPEN? CLOSED? BOUNDED?

$$(a) f(x, y) = \sqrt{y-x}$$

$$(b) f(x, y) = \ln(x^2 + y^2 - 1)$$

$$\int_1^2 \left[\frac{1}{2}x^2 - \frac{1}{52} \left(\frac{1}{2} - 1 \right) \right] dx =$$

$$\int_1^2 \left[\frac{1}{2}x^2 - 0 \right] dx =$$

$$\int_0^{\pi/2} \cos^2(x) \sin(x) dx = \int_0^{\pi/2} (1 - \sin^2(x)) \sin(x) dx = \int_0^{\pi/2} \sin(x) dx - \int_0^{\pi/2} \sin^3(x) dx$$

$$\int_0^{\pi/2} \sin(x) dx = \left[-\cos(x) \right]_0^{\pi/2} = 1$$

$$\int_0^{\pi/2} \sin^3(x) dx = \int_0^{\pi/2} \sin(x) (1 - \cos^2(x)) dx = \int_0^{\pi/2} \sin(x) dx - \int_0^{\pi/2} \sin(x) \cos^2(x) dx$$

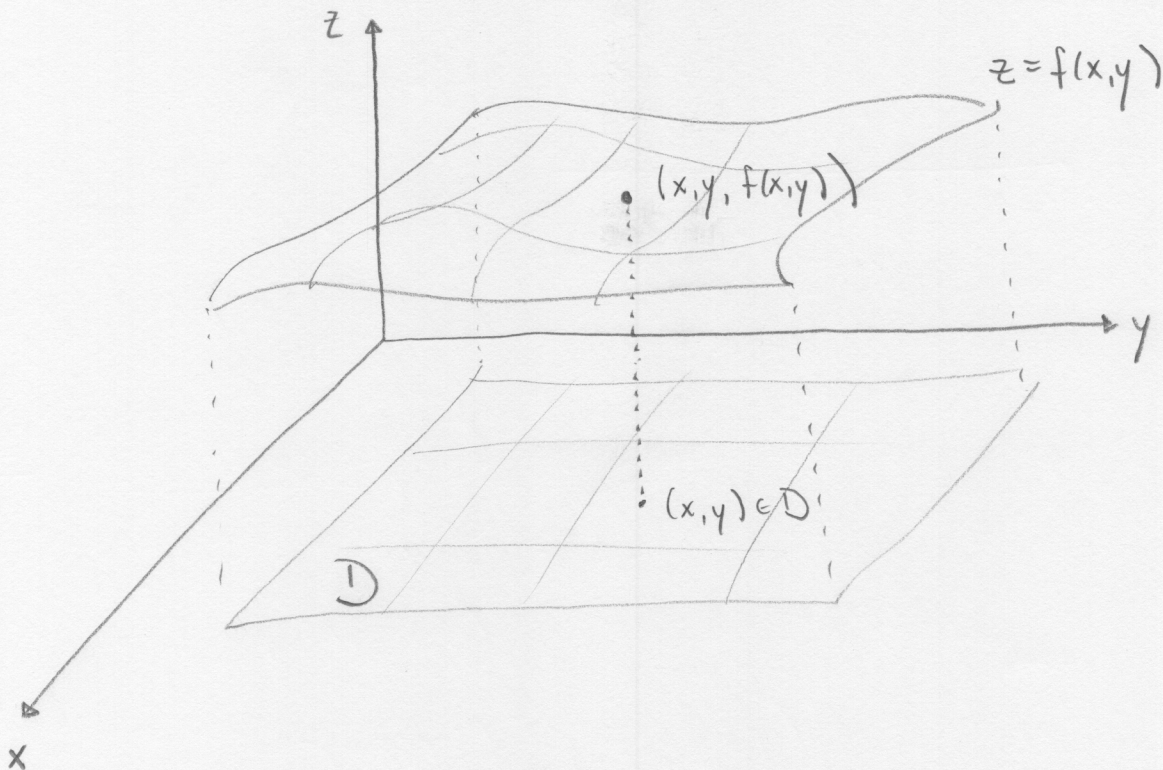
$$\int_0^{\pi/2} \sin(x) \cos^2(x) dx = \left[-\frac{1}{3} \cos^3(x) \right]_0^{\pi/2} = \frac{1}{3}$$

GRAPHS : 2 VAR.

Def: Given function f of 2 variables with domain D ,

its GRAPH is the set of all points (x, y, z) in $\mathbb{R}^3$

s.t. $z = f(x, y)$ & $(x, y) \in D$. (also called a SURFACE)



Def: Given a constant c , the set of points in D s.t.

$f(x, y) = c$ is called a LEVEL CURVE of f .

ex. consider $f(x, y) = x^2 + 2y^2$

Sketch level curves for $c = 0, 1, 2, \dots$

Sketch graph / surface $z = f(x, y)$.

ex. Find Eq of level curve of $f(x, y) = \frac{2y-1}{3x+4y-2}$
through $(1, 1)$.

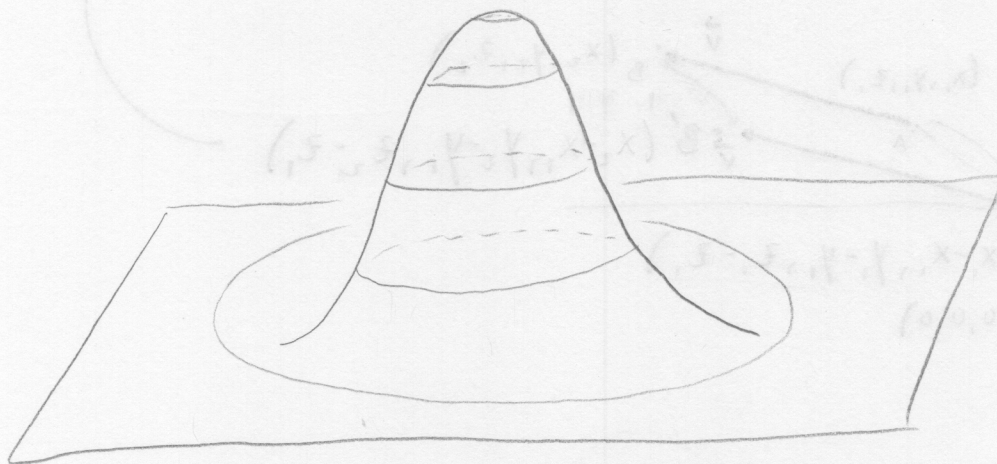
CONSIDER A MOUNTAIN AS A SURFACE.

IF IT WERE THE GRAPH OF A FUNCTION $f(x, y)$

$$\text{i.e. } z = f(x, y)$$

THEN THE LEVEL CURVES ARE THE PATHS THAT

HAVE CONSTANT ELEVATION - NOT UPHILL OR DOWNHILL.



GRAPHS : 3 var

Def: Given a constant C , the set of all points $(x, y, z) \in D$

s.t. $f(x, y, z) = C$ is called a level surface of f .



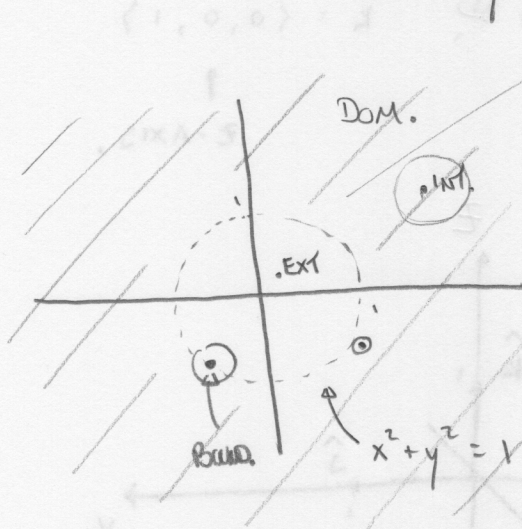
THE BEST WE CAN DO ☹️

ex. FIND EQ OF LEVEL SURFACE OF $f(x, y, z) = \ln(x^2 + y^2 + z^2)$

PASSING THROUGH $(-1, 2, 1)$.

$$\text{Dom: } x^2 + y^2 - 1 > 0$$

$$x^2 + y^2 > 1$$



$$\text{Boundary: } x^2 + y^2 = 1$$

↑
Not in Dom

∴ Dom is not closed.

Is domain open?

Yes.

Bounded? No.

TOPOLOGY

THE STUDY OF OPEN SETS.

ex. FIND DOMAIN & FIND BOUNDARY POINTS OF DOMAIN.

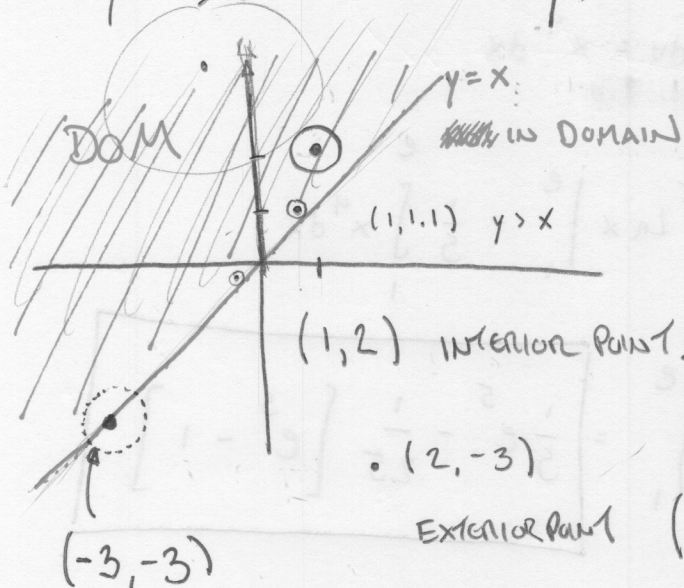
IS THE DOMAIN OPEN? CLOSED? BOUNDED?

(a) $f(x,y) = \sqrt{y-x}$ Dom $y \geq x$

(b) $f(x,y) = \ln(x^2 + y^2 - 1)$

$f(x,y) = \frac{1}{\sqrt{y-x}}, y \neq x$

(a) $y - x \geq 0$ DOMAIN $y \geq x$



BOUNDARY? LINE $y=x$

DOES THE DOMAIN CONTAIN ITS BOUNDARY? YES!

DOM CLOSED? YES!

SAME QUESTION!

EXTERIOR POINT (INTERIOR POINT OF THE COMPLEMENT)

BOUNDARY POINT

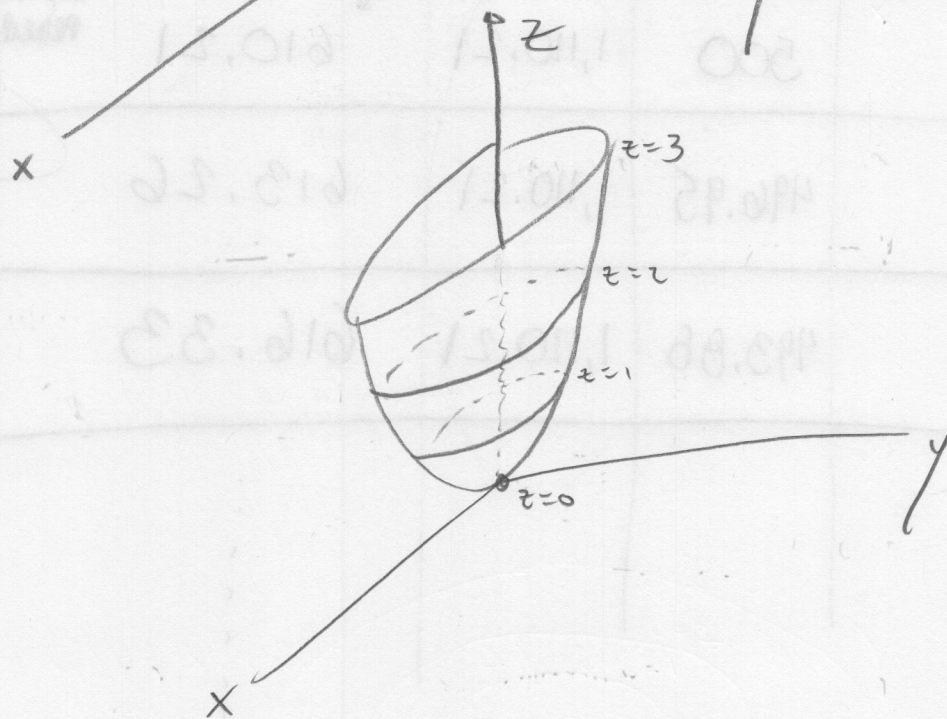
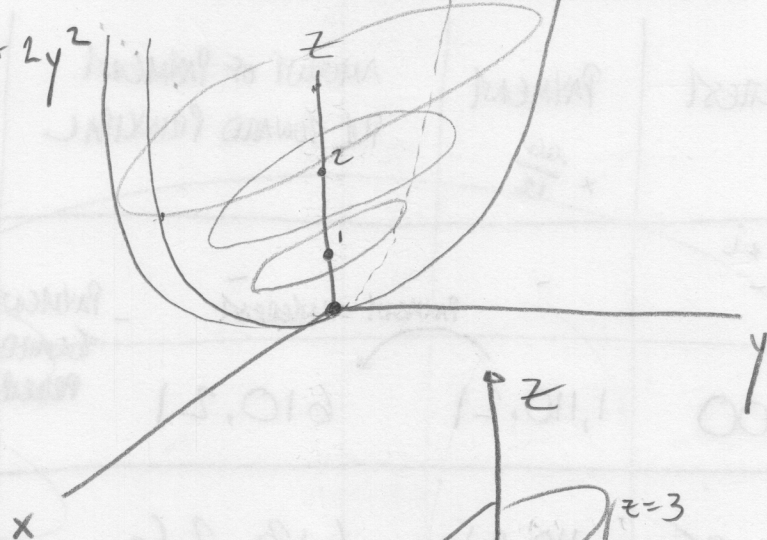
Dom BOUNDED? No.

IS THE DOMAIN OPEN?

No (BOUNDARY POINTS ARE IN DOM, & THEY ARE NOT INTERIOR POINTS)

USE THE LEVEL CURVES TO HELP YOU SKETCH THE GRAPH

$$z = x^2 + 2y^2$$



ex. FIND EQ. OF THE LEVEL CURVE FOR

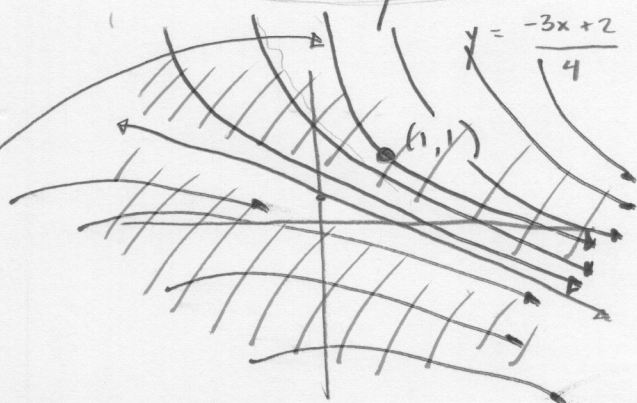
$$f(x, y) = \frac{2y-1}{3x+4y-2} = 0$$

THROUGH $(1, 1)$.

$$f(x, y) = c \text{ s.t. } (1, 1) \text{ is a sol'n.}$$

$$f(1, 1) = \frac{1}{5}$$

$$\boxed{\frac{2y-1}{3x+4y-2} = \frac{1}{5}}$$



LEVEL CURVES FOR FUNC. OF 3 VAR'S

Def. Given $f(x, y, z)$ with domain D ,

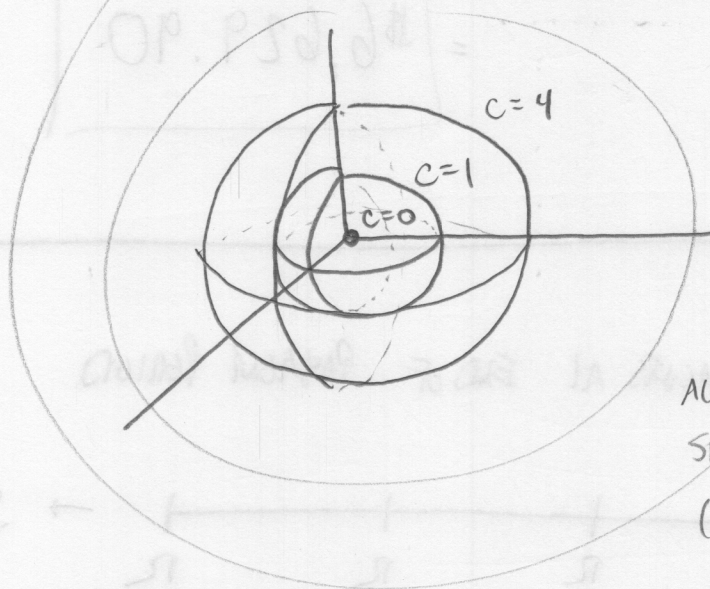
THE SET OF ALL POINTS (x, y, z) s.t.

$$f(x, y, z) = C, \text{ FOR SOME CONSTANT,}$$

IS CALLED A LEVEL SURFACE FOR f .

e.g. $f(x, y, z) = x^2 + y^2 + z^2 = C$

FAMILY OF LEVEL CURVES.



$$C < 0 \Rightarrow \text{NO SOLNS}$$

$$C = 0 \Rightarrow (0, 0, 0)$$

$$C = 1$$

ALL LEVEL SURFACES ARE
SPHERES WITH CENTER
 $(0, 0, 0)$.