## §10.6 ALTERNATING SERIES & CONDITIONAL CONVERGENCE

**Def:**

A SERIES $\sum_{n=1}^{\infty} a_n$ IS <u>ALTERNATING</u> IF IT HAS THE FORM

$$\boxed{\pm \sum_{n=1}^{\infty} (-1)^{n+1} u_n}, \quad u_n > 0$$

$u_n = |a_n|$

FIRST TERM CAN BE EITHER POS. OR NEG.

ALTERNATING POSITIVE / NEGATIVE

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**ALTERNATING SERIES TEST:**

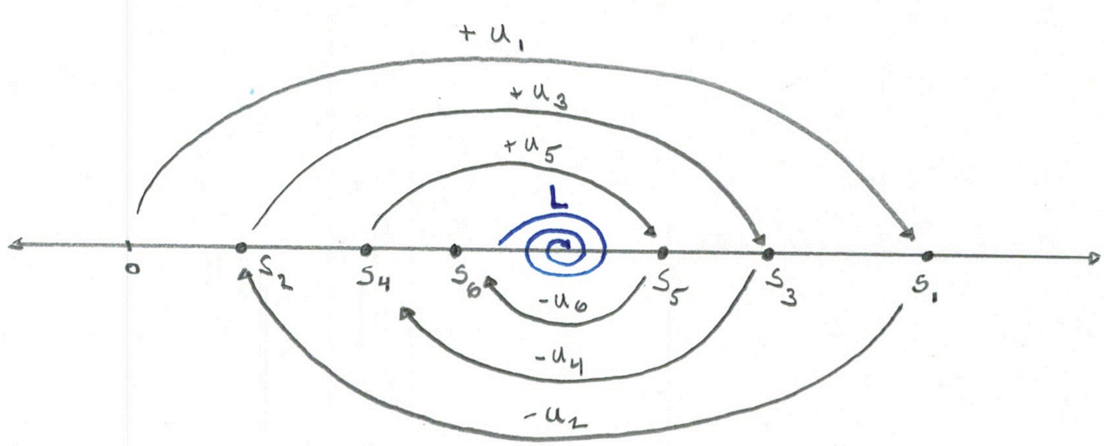
AN ALTERNATING SERIES $\pm \sum_{n=1}^{\infty} (-1)^{n+1} u_n$, $u_n > 0$

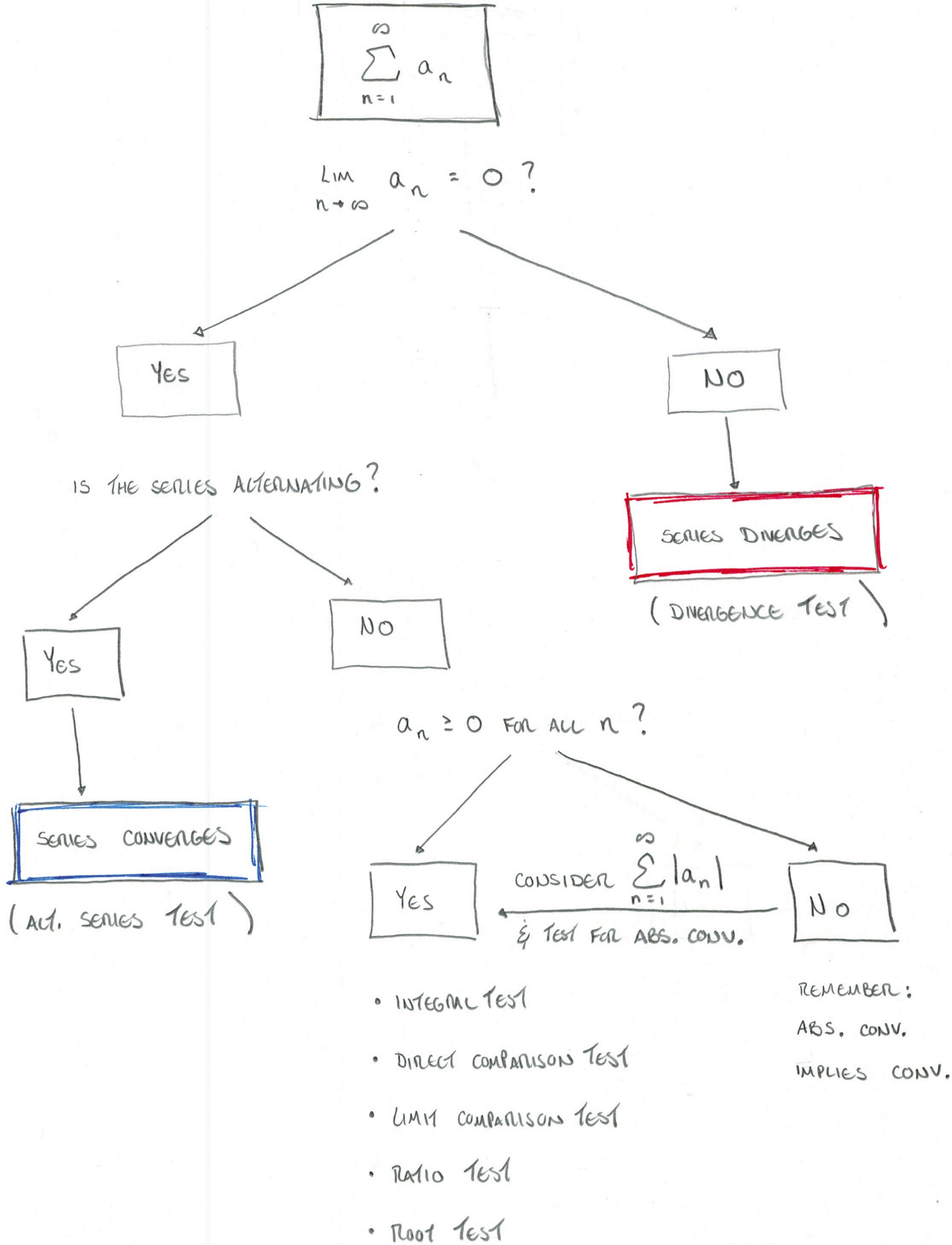
CONVERGES IF

1. $u_n \geq u_{n+1}$ EVENTUALLY

   FOR ALL $n \geq N$, FOR SOME INTEGER $N$

2. $\lim_{n \to \infty} u_n = 0$.





$\sum_{n=1}^{\infty} a_n$
$\lim_{n\to\infty} a_n = 0$ ?
YES
NO
SERIES DIVERGES
( DIVERGENCE TEST )
IS THE SERIES ALTERNATING ?
YES
NO
SERIES CONVERGES
( ALT. SERIES TEST )
$a_n \geq 0$ FOR ALL $n$ ?
YES
NO
CONSIDER $\sum_{n=1}^{\infty} |a_n|$
& TEST FOR ABS. CONV.
• INTEGRAL TEST
• DIRECT COMPARISON TEST
• LIMIT COMPARISON TEST
• RATIO TEST
• ROOT TEST
REMEMBER:
ABS. CONV.
IMPLIES CONV.

**ex 1.** ALTERNATING HARMONIC SERIES

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

CONVERGES. (PLOT.)

**Def:** A SERIES THAT CONVERGES BUT DOES NOT CONVERGE ABSOLUTELY IS CALLED <u>CONDITIONALLY CONVERGENT</u> (IT <u>CONVERGES CONDITIONALLY</u>).

**ex. 2** CONVERGE OR DIVERGE?

$$\sum_{n=1}^{\infty} (-1)^n \frac{10^n}{(n+1)!}$$

**ex. 3** CONVERGE OR DIVERGE?

$$\sum_{n=1}^{\infty} (-1)^n \ln\left(1 + \frac{1}{n}\right)$$

**ex. 4** CONVERGE OR DIVERGE?

$$\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n}{10}\right)^n$$

**ex. 5** CONVERGE OR DIVERGE?

$$\sum_{n=1}^{\infty} \frac{(-2)^{n+1}}{n + 5^n}$$

# ALTERNATING p-SERIES TEST

$$\lim_{n \to \infty} n^p = \infty \quad \text{IF} \quad p > 0$$

$$\lim_{n \to \infty} \frac{1}{n^p} = 0 \quad \text{IF} \quad p > 0$$

THEREFORE, $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^p}$ CONVERGES IF $p > 0$.

MORE PRECISELY,

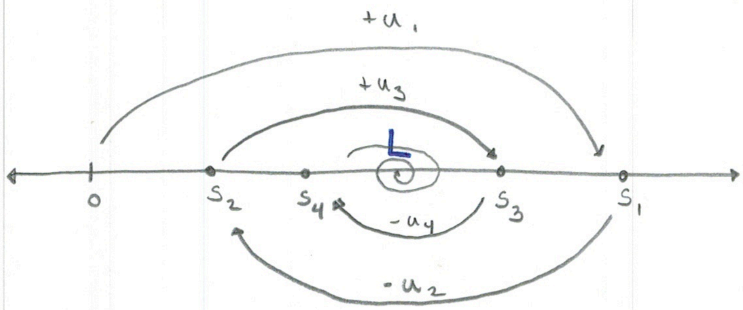
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^p} \begin{cases} \bullet \text{ DIVERGES IF } p \leq 0 \\ \bullet \text{ CONVERGES CONDITIONALL} \\ \quad \text{IF } 0 < p \leq 1 \\ \bullet \text{ CONVERGES ABSOLUTELY} \\ \quad \text{IF } p > 1 \end{cases}$$

## ALTERNATING SERIES ESTIMATION THEOREM

SUPPOSE AN ALTERNATING SERIES CONVERGES.

$$\sum_{n=1}^{\infty} (-1)^{n+1} u_n = L$$

WE CAN APPROXIMATE $L \approx S_n = u_1 - u_2 + u_3 - \ldots + (-1)^{n+1} u_n$




NOTE:

- IF LAST TERM IS POSITIVE, THEN THIS IS AN OVERESTIMATE
- IF LAST TERM IS NEGATIVE THEN THIS IS AN UNDERESTIMATE.

AND THE ERROR IS BOUNDED BY $u_{n+1}$.

**ex. 6**

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = L$$

$S_{30} = 0.67676$

OVERESTIMATE OR UNDEREST.?

MAXIMUM ERROR?