

Part I: Answer all 7 questions (10 points each)

1. (a) Give an equation for the plane through the following three points.

$$A = (1, 1, 1) \quad B = (-3, 2, 5) \quad C = (4, -1, 6)$$

- (b) The line l has the following parametric equations.

$$x = 4 - 2t, \quad y = 3 - 5t, \quad z = -3 - 3t \tag{l}$$

Determine whether each of the following planes is parallel to l , perpendicular to l , or neither .

i. $x + 5y - 4z = -9$

ii. $4x + 10y + 6z = 16$

iii. $-25x - 11y + 35z = 23$

- (c) Find parametric equations for the line through $(2, 4, 6)$ that is perpendicular to the plane

$$x - y + 3z = 7.$$

2. (a) Give a unit tangent vector to the following parametric curve at the point $(4, 2, 7)$.

$$\mathbf{r}(t) = \langle t^3 + 3t, t^2 + 1, 3t + 4 \rangle$$

- (b) Give an equation for the tangent plane to the following surface at the point $(0, 0, 3)$.

$$z = \frac{2x + 3}{4y + 1}$$

3. For each of the following, evaluate the limit or show that it does not exist.

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{5y^4 \cos^2 x}{x^4 + y^4}$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$

4. Suppose that over a certain region of space the electrical potential V is given by

$$V(x, y, z) = 5x^2 - 3xy + xyz.$$

(a) Find the rate of change of V at the point $P = (3, 4, 5)$ in the direction of the vector $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.

(b) In which direction does V change most rapidly at P ?

(c) Now suppose that

$$x = s + t, \quad y = 2s - 3t, \quad z = st^2.$$

Find $\frac{\partial V}{\partial t}$ at the point $(s, t) = (1, 1)$.

5. Find all critical points for the function

$$f(x, y) = y^2 - 2y \cos x$$

and classify each one as a local maximum, local minimum, or saddle point.

6. (a) Find the volume of the solid enclosed by the cylinder $y = x^2$ and the planes $z = 0$ and $y + z = 1$.

(b) Find the volume of the solid above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 4$.

7. For each of the following series, state whether it is absolutely convergent, conditionally convergent, or divergent. Name a test which supports each conclusion, and show the work to apply the test.

(a)
$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

(b)
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{5n+1}$$

(c)
$$\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1} \right)^n$$

Part II: Answer 3 out of 5 questions (10 points each)

8. Find the area of the following parametric surface.

$$\mathbf{r}(u, v) = \langle u \cos v, u \sin v, v \rangle, \quad u^2 + v^2 \leq 1$$

9. (a) In spherical coordinates, the cone $9z^2 = x^2 + y^2$ has equation $\phi = c$. Find c .

(b) Find

$$\iiint_R (x^2 + y^2 + z^2)^{3/2} dV,$$

where R is the region inside the sphere $x^2 + y^2 + z^2 = 3$ and inside the cone $z = \frac{\sqrt{x^2 + y^2}}{3}$.

10. Let

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

- (a) Show that $f_x(0, 0)$ and $f_y(0, 0)$ both exist. Note that you must use the definition of partial derivatives as limits.

- (b) Explain why f is not differentiable at $(0, 0)$.

11. (a) Use differentials to estimate the amount of metal in a closed cylindrical can that is 10 cm high and 4 cm in diameter if the metal in the top and bottom is 0.1 cm thick and the metal in the sides is 0.05 cm thick.

- (b) Find the mass of the lamina that occupies the triangular region enclosed by the following lines and has density $\delta(x, y) = x^2$.

$$x = 0, \quad y = x, \quad 2x + y = 6$$

12. For each of the following functions, find the maximum and minimum values of the function on the circular disk $x^2 + y^2 \leq 1$. Do this by first sketching the level curves and gradients.

(a) $f(x, y) = x + y + 1$

(b) $f(x, y) = x^2 + 2y^2$