Name: * ANSWER KEY * 11/25/2014

Math 203 Calculus III **Quiz 3**

Please show all work and **box your final answers**. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

1. (8 points) Find all critical points of the function

$$f(x,y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$$

and classify each as either a local maximum, local minimum, or saddle point.

$f_x = 6xy - 12x = 0$

$6x(y-2) = 0$

$x = 0$ or $y = 2$

$f_y = 3y^2 + 3x^2 - 12y = 0$

$x = 0 \longrightarrow 3y^2 - 12y = 3y(y-4) = 0$

$\longrightarrow \boxed{(0,0),\ (0,4)}$

$y = 2 \longrightarrow 12 + 3x^2 - 24 = 3x^2 - 12 = 0$

$3(x^2-4) = 3(x+2)(x-2) = 0$

$\longrightarrow \boxed{(2,2),\ (-2,2)}$

SECOND DERIVATIVES TEST

$f_{xx} = 6y - 12$ $\quad f_{yy} = 6y - 12$ $\quad f_{xy} = 6x$

$D(0,0) = (-12)(-12) - (0)^2 > 0,\ f_{xx}(0,0) = -12 < 0 \longrightarrow \boxed{(0,0) \text{ LOCAL MAX}}$

$D(0,4) = (12)(12) - (0)^2 > 0,\ f_{xx}(0,4) = 12 > 0 \longrightarrow \boxed{(0,4) \text{ LOCAL MIN}}$

$D(2,2) = (0)(0) - (12)^2 < 0 \longrightarrow \boxed{(2,2) \text{ SADDLE POINT}}$

$D(-2,2) = (0)(0) - (-12)^2 < 0 \longrightarrow \boxed{(-2,2) \text{ SADDLE POINT}}$

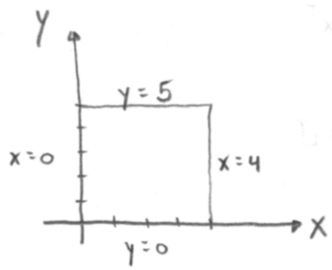
2. (8 points) Find the absolute maximum and absolute minimum value(s) of f over the domain D, where

$$f(x,y) = 4x + 6y - x^2 - y^2,$$

$$D = \{(x,y) : 0 \le x \le 4, 0 \le y \le 5\}.$$

CRITICAL PNT(S): $f_x = 4 - 2x$ $\quad$ $f_y = 6 - 2y$ $\quad \longrightarrow (2,3)$

$x = 2$ $\quad$ $y = 3$



$y = c \quad (c = 0 \text{ or } 5)$

$f(x,c) = 4x - x^2 + 6c - c^2$

$\frac{d}{dx} \longrightarrow 4 - 2x = 0 \longrightarrow x = 2$

$\longrightarrow (2,0)\ ,\ (2,5)$

$x = c \quad (c = 0 \text{ or } 4)$

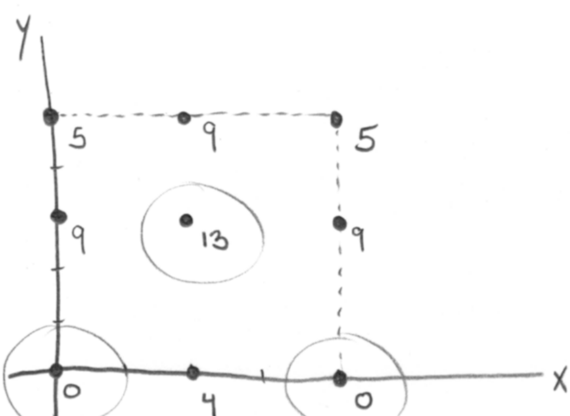
$f(c,y) = 6y - y^2 + 4c - c^2$

$\frac{d}{dy} \longrightarrow 6 - 2y = 0 \longrightarrow y = 3$

$\longrightarrow (0,3)\ ,\ (4,3)$

(x,y)	$f(x,y)$
$(2,3)$	$8 + 18 - 4 - 9 = 13$
$(2,0)$	$8 - 4 = 4$
$(2,5)$	$8 + 30 - 4 - 25 = 9$
$(0,3)$	$18 - 9 = 9$
$(4,3)$	$16 + 18 - 16 - 9 = 9$
$(0,0)$	0
$(4,0)$	$16 - 16 = 0$
$(0,5)$	$30 - 25 = 5$
$(4,5)$	$16 + 30 - 16 - 25 = 5$

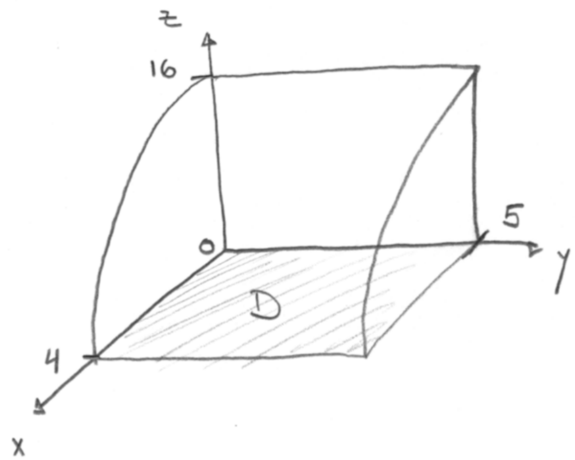
CORNER PNTS.



ABS. MAX VALUE 13

ABS. MIN VALUE 0

3. (8 points) Find the volume of the solid in the first octant bounded by the cylinder $z = 16 - x^2$ and the plane $y = 5$.



$$V = \iint_D 16 - x^2 \, dA$$

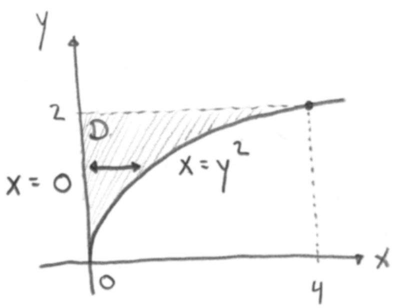
$$= \int_0^5 \int_0^4 16 - x^2 \, dx \, dy = \int_0^5 16x - \frac{1}{3}x^3 \Big|_0^4 \, dy$$

$$= \frac{128}{3} \int_0^5 dy = \frac{128}{3} \cdot 5 = \boxed{\frac{640}{3}}$$

4. (8 points) Evaluate the integral by reversing the order of integration.

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dy\, dx$$

Hint: First sketch the region D over which we are integrating.



$$\int_0^2 \int_0^{y^2} \frac{1}{y^3+1}\, dx\, dy$$

$$= \int_0^2 \frac{x}{y^3+1} \Big|_0^{y^2} dy$$

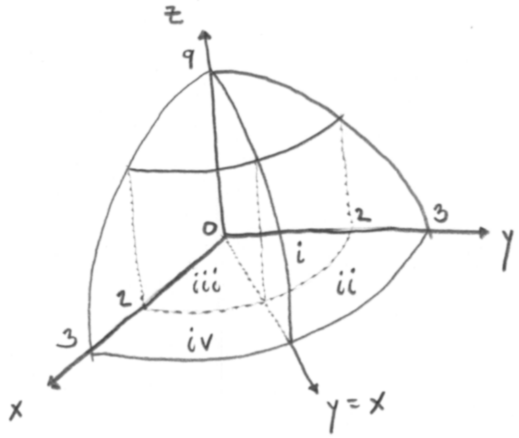
$$= \int_0^2 \frac{y^2}{y^3+1}\, dy$$

LET $u = y^3 + 1$

$du = 3y^2$

$$\rightarrow \frac{1}{3}\int_1^9 \frac{1}{u}\, du = \frac{1}{3} \operatorname{Ln} u \Big|_1^9 = \boxed{\frac{1}{3} \operatorname{Ln} 9}$$

5. (8 points) Find the volume of the solid contained in the first octant which is bounded by the cylinder $x^2 + y^2 = 4$, the paraboloid $z = 9 - x^2 - y^2$, and the planes $x = 0$, $z = 0$, and $y = x$.



THIS IS A BAD QUESTION! SORRY!

(IT IS QUESTION #5 FROM FALL 2005 FINAL EXAM)

IT'S BAD BECAUSE IT'S NOT CLEAR IF IT WANTS THE VOLUME OF THE SOLID ABOVE REGION i, ii, iii, OR iv.

BY SYMMETRY, VOLUME OF SOLID ABOVE REGION i = iii.

" " " ii = iv.

<u>VOL. ABOVE i:</u>

$$\iint_{i} 9-x^2-y^2 \, dA = \int_0^{\pi/4}\int_0^{2} (9-r^2)r \, dr \, d\theta$$

$$= \frac{\pi}{4}\left(\frac{9}{2}r^2 - \frac{1}{4}r^4\right)\Bigg|_0^2 = \frac{\pi}{4}(18-4) = \boxed{\frac{7\pi}{2}}$$

<u>VOL. ABOVE ii:</u>

$$\iint_{ii} 9-x^2-y^2 \, dA = \int_0^{\pi/4}\int_2^{3} (9-r^2)r \, dr \, d\theta$$

$$= \frac{\pi}{4}\left(\frac{9}{2}r^2 - \frac{1}{4}r^4\right)\Bigg|_2^3 = \frac{\pi}{4}\left[\frac{9}{2}(9-4) - \frac{1}{4}(81-16)\right]$$

$$= \frac{\pi}{4}\left(\frac{45}{2} - \frac{65}{4}\right) = \frac{\pi}{4}\cdot\frac{25}{4} = \boxed{\frac{25\pi}{16}}$$