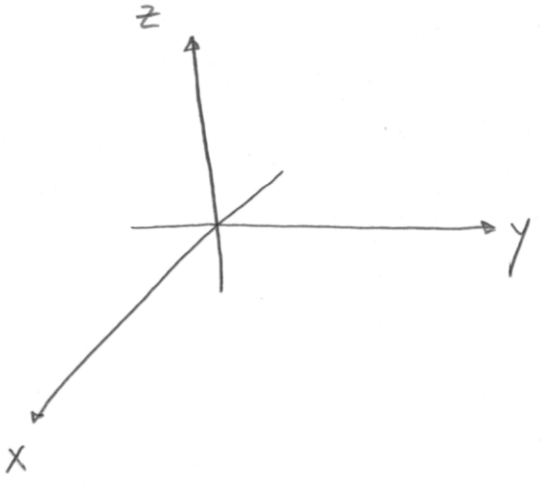
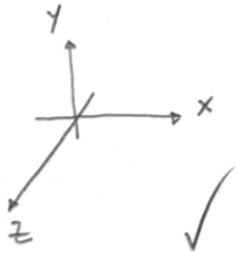
§ 10.1 THREE-DIMENSIONAL COORDINATE SYSTEMS



RIGHT-HAND RULE: GRIP THE z-AXIS & CURL FINGERS FROM x-AXIS TO y-AXIS, THUMB POINTS IN POSITIVE z DIRECTION).



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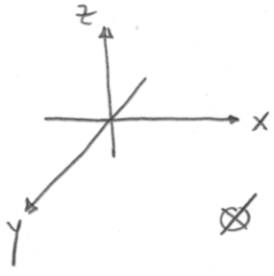

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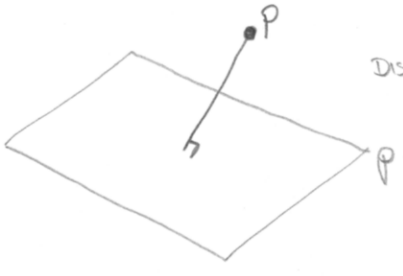
COORDINATE PLANES: xy-, yz-, xz-PLANES.



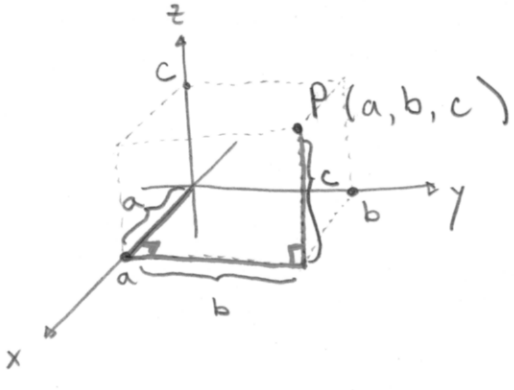
DISTANCE FROM POINT P TO PLANE P.

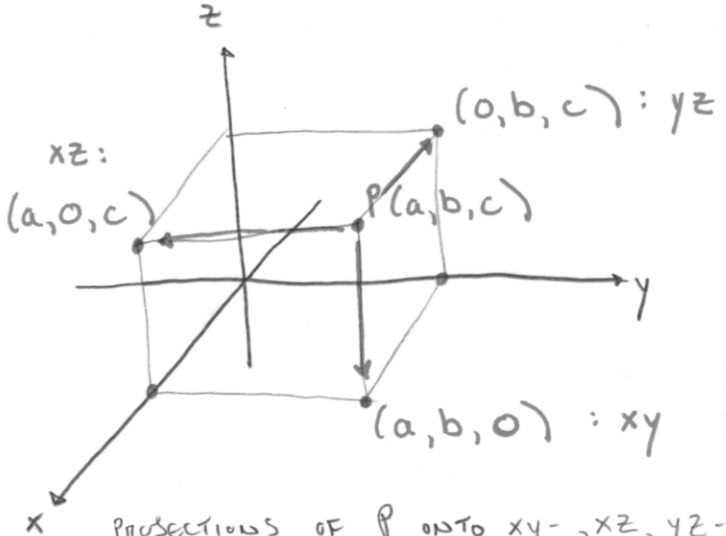
GIVEN A POINT P IN SPACE, LET a = DIST. TO yz-PLANE

b = DIST TO xz-PLANE

c = DIST TO xy-PLANE

a, b, c ARE CALLED COORDINATES OF POINT P, AND DETERMINE THE LOCATION OF P AS FOLLOWS





PROJECTIONS OF P ONTO xy-, xz-, yz-PLANES

PLOT POINTS

GIVEN AN EQUATION IN x, y, AND z, ITS GRAPH IS THE SET OF ALL POINTS (x, y, z) WHICH SATSFY THE EQUATION.

e.g. GIVEN EQ: $x + y = z$

NAME SOME POINTS ON ITS GRAPH.

e.g. $x^2 + y^2 = z^2$

...

RECALL THAT THE GRAPH OF AN EQ IN x & y IS A CURVE (IN 2-DIM. PLANE).

NOW, THE GRAPH OF AN EQ IN x, y, z IS A SURFACE (IN 3-DIM. SPACE).

[COMP. GRAPH]

NOTE THAT THE xy-PLANE IS GRAPH OF $z = 0$,

yz-PLANE IS GRAPH OF $x = 0$,

xz-PLANE IS GRAPH OF $y = 0$.

WHAT IF 0 IS REPLACED BY ANOTHER CONSTANT, c?

[COMP. GRAPH]

DISTANCE FORMULA (3D)

GIVEN $P_1(x_1, y_1, z_1)$ AND $P_2(x_2, y_2, z_2)$

THE DISTANCE BETWEEN P_1 AND P_2 IS

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

[PROOF]

e.g. FIND DISTANCE BETWEEN $P_1(6, -5, 9)$

$P_2(-2, 4, -3)$

$8^2 + 9^2 + 12^2 = 17^2$

[WIKI: PYTHAGOREAN QUADRUPLE]

e.g. FIND DISTANCE BETWEEN $P = (2,3,4)$ AND $X = (x,y,z)$.

$$d = \sqrt{(2-x)^2 + (3-y)^2 + (4-z)^2}.$$

$$\downarrow$$

$$d^2 = (2-x)^2 + (3-y)^2 + (4-z)^2$$

$\downarrow$ NOW FIX d. $(d=1)$

$$1 = (2-x)^2 + (3-y)^2 + (4-z)^2.$$

THIS IS AN EQ IN x,y,z. WHAT SHAPE IS ITS GRAPH?

[COMP GRAPH]

EQ OF A SPHERE

AN EQ FOR SPHERE WITH CENTER (a,b,c) AND RADIUS r IS

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2.$$

NOTE, IF CENTER IS ORIGIN, THEN WE HAVE

$$x^2 + y^2 + z^2 = r^2$$

e.g. SHOW THAT $x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0$ REPRESENTS A SPHERE, FIND ITS RADIUS & CENTER.

e.g. FIND INEQUALITY TO DESCRIBE SOLID BALL CENTERED AT ORIGIN, RADIUS R.

e.g. FIND INEQUALITY TO DESCRIBE REGION BETWEEN SPHERES OF RADIUS r & R $(r<R)$.

(JUST UPPER HEMISPHES?).