Name: ANSWER KEY 6/12/2018

Math 202 Calculus II Quiz 1

Please show all work and box your final answers. If you need more room, you may use the backs of the pages. Calculators are not allowed. Good luck!

1. Find $(f^{-1})'(a)$, where $f(x) = 5x^3 + 5x + 12$ and $a = 12$.

$$(f^{-1})'(12) = \frac{1}{f'(\underbrace{f^{-1}(12)})}$$

$f^{-1}(12) = 0$ BECAUSE $f(0) = 12$

$f'(x) = 15x^2 + 5$

$$\therefore (f^{-1})'(12) = \frac{1}{f'(0)} = \boxed{\frac{1}{5}}$$

2. Find the domain of $f(x) = \ln(\underbrace{\ln(\ln x)})$.

SINCE DOM$(\ln x)$ IS $(0, \infty)$,

THE INPUT INTO THE OUTER $\ln$ MUST BE POSITIVE.

i.e. $\ln(\ln x) > 0$

$\Rightarrow e^{\ln(\ln x)} > e^0$

$\ln x > 1$

$\Rightarrow e^{\ln x} > e^1$

$x > e$

$$\boxed{(e, \infty)}$$

3. Differentiate the following functions.

(a) $y = \ln \frac{1+2x}{3-4x}$

$y = \ln(1+2x) - \ln(3-4x)$

$$\boxed{y' = \frac{2}{1+2x} + \frac{4}{3-4x}}$$

(b) $y = 9^x$

$\left(y = (e^{\ln 9})^x = e^{(\ln 9)x} \right)$

$$\boxed{y' = 9^x \ln 9}$$

(c) $y = x^{\sin x}$ (LOGARITHMIC DIFFERENTIATION)

$\ln y = \sin x \ln x$

$\frac{1}{y} y' = \cos x \ln x + \frac{\sin x}{x}$

$y' = y\left(\cos x \ln x + \frac{\sin x}{x} \right) = \boxed{x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x} \right)}$

4. Evaluate the following integrals.

(a) $\int \frac{e^x+2}{3e^x}\,dx$

LET $u = -x$
$du = -dx$
$-du = dx$

$\leadsto \frac{2}{3}\int -e^{u}\,du \ldots$

$$= \int \frac{e^x}{3e^x}\,dx + \int \frac{2}{3e^x}\,dx = \frac{1}{3}\int dx + \frac{2}{3}\int e^{-x}\,dx$$

$$= \boxed{\frac{1}{3}x - \frac{2}{3}e^{-x} + c}$$

(b) $\int_2^4 \frac{2^{x-1}}{2^{x-1}+1}\,dx$

LET $u = 2^{x-1}+1$
$du = 2^{x-1}\ln 2\,dx$
$\frac{1}{\ln 2}\,du = 2^{x-1}\,dx$

$$\int_{x=2}^{x=4} \xrightarrow{u=2^{x-1}} \int_{u=3}^{9}$$

$$\leadsto \frac{1}{\ln 2}\int_3^9 \frac{1}{u}\,du = \frac{1}{\ln 2}\ln|u|\Big|_3^9$$

$$= \frac{\ln 9 - \ln 3}{\ln 2} = \frac{\ln\left(\frac{9}{3}\right)}{\ln 2} = \boxed{\frac{\ln 3}{\ln 2} = \log_2 3}$$