Name: ANSWER KEY **2/22/2018**

Math 202 Calculus II **Quiz 1**

All electronic devices must be turned off and put away (e.g. cellphones, calculators, etc.). Answer all of the following questions. Show all of your work and put boxes around your final answers. If you need more room, you may continue your work on the backs of the pages. Good luck!

1. The following functions are one-to-one.

$$f(x) = \frac{x-1}{2x+3}, \qquad g(x) = x^3 + 3\sin x + 2\cos x$$

4 (a) Find $f^{-1}(x)$.

$$y = \frac{x-1}{2x+3}$$

$$2xy + 3y = x - 1$$

$$x(2y-1) = -3y - 1$$

$$x = \frac{-3y-1}{2y-1} \Rightarrow \boxed{f^{-1}(x) = \frac{-3x-1}{2x-1}}$$

4 (b) Find $(g^{-1})'(2)$.

SINCE $g(0) = 2$, $g^{-1}(2) = 0$.

INVERSE FUNCTION THM SAYS

$$(g^{-1})'(2) = \frac{1}{g'(g^{-1}(2))} = \frac{1}{g'(0)}$$

SINCE $g'(x) = 3x^2 + 3\cos x - 2\sin x$,

$$g'(0) = 3$$

SO $\boxed{(g^{-1})'(2) = \frac{1}{3}}$

2. Consider the following functions.

$$f(x) = \ln x, \qquad g(x) = e^x$$

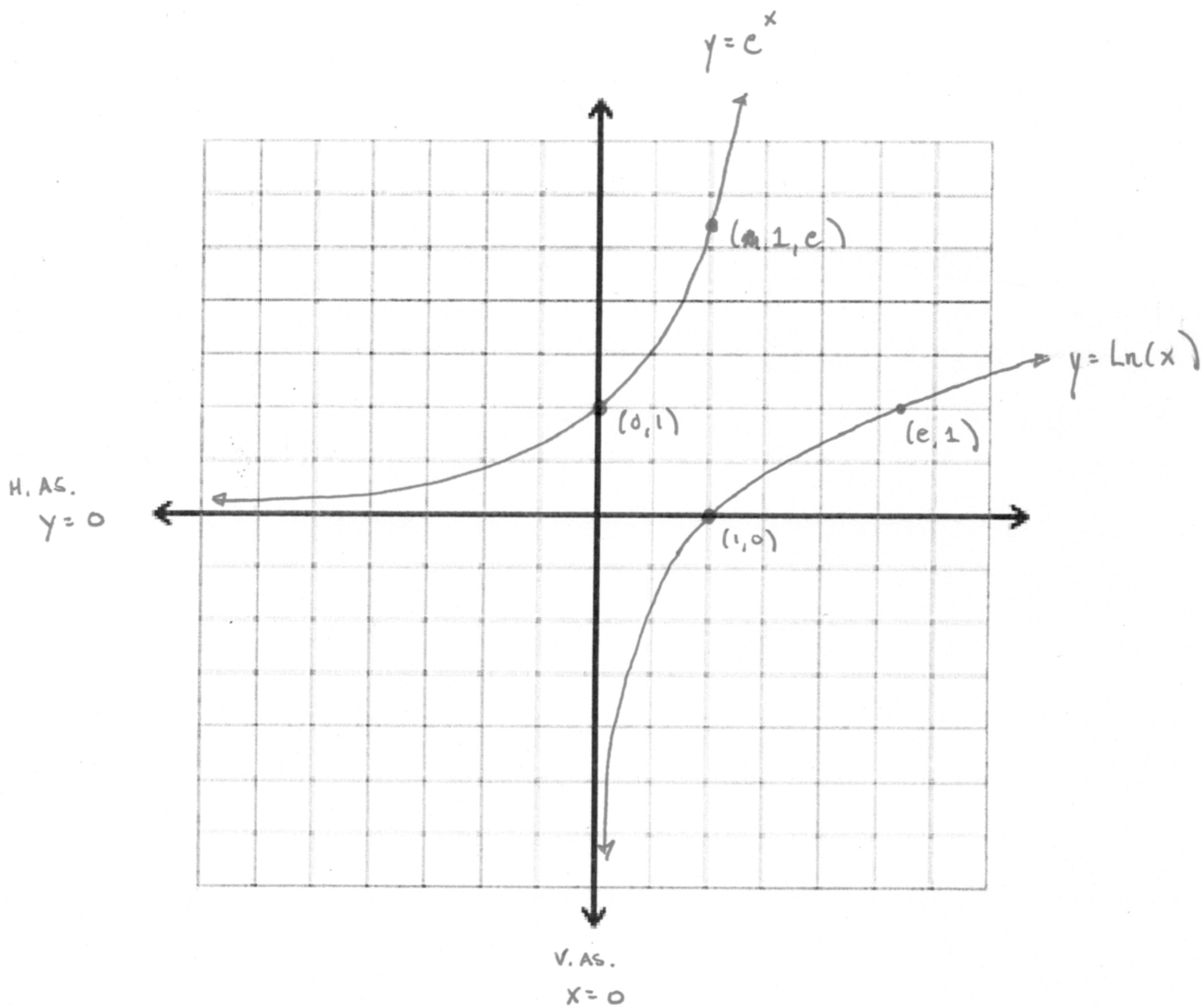
2 (a) State the domain and range for f.

DOM$(f) = (0, \infty)$ RAN$(f) = (-\infty, \infty)$

2 (b) State the domain and range for g.

DOM$(g) = (-\infty, \infty)$ RAN$(g) = (0, \infty)$

6 (c) Below, sketch the graphs $y = f(x)$ and $y = g(x)$. Label any/all horizontal/vertical asymptotes, x/y intercepts, and at least one additional point on the graph that is not an intercept.
Note: you can use whatever scale you like for the axes. For example, you can let each square have side-length 0.5.



4 3. Use the Laws of Logarithms to expand and simplify the following expression.

$$\ln\left(\frac{(x^2+4)^6\sin^2 x}{\sqrt[3]{e^x}}\right)$$

$\ln\left((x^2+4)^6\right) + \ln(\sin^2 x) - \ln\left(\sqrt[3]{e^x}\right)$

$6\ln(x^2+4) + 2\ln(\sin x) - \frac{1}{3}\ln(e^x)$

$\boxed{6\ln(x^2+4) + 2\ln(\sin x) - \frac{1}{3}x}$

4 4. Calculate the following limit. If it does not exist, write D.N.E.

$$\lim_{x\to 2^-} e^{x/(x-2)}$$

NOTE : $\lim_{x\to 2^-} \frac{x}{x-2}$ $\left(\text{THINK } \frac{2}{0^-}\right)$ $= -\infty$

SO $\lim_{x\to 2^-} e^{\frac{x}{x-2}} = \lim_{w\to -\infty} e^w = \boxed{0}$

5. Differentiate the following functions.

4 (a) $f(x) = \ln\left(\frac{a-x}{a+x}\right)$

$$= \ln(a-x) - \ln(a+x)$$

$$\boxed{f'(x) = \frac{1}{a-x}(-1) - \frac{1}{a+x}, \text{ or } = \frac{-2a}{a^2-x^2}}$$

4 (b) $g(x) = \frac{\sin^2 x \tan^4 x}{(x^2+1)^2}$

Use logarithmic differentiation.

$$\ln(g(x)) = 2\ln(\sin x) + 4\ln(\tan x) - 2\ln(x^2+1)$$

$$\frac{1}{g(x)} g'(x) = \frac{2\cos x}{\sin x} + \frac{4\sec^2 x}{\tan x} - \frac{4x}{x^2+1}$$

$$\therefore g'(x) = g(x)\left(\frac{2\cos x}{\sin x} + \frac{4\sec^2 x}{\tan x} - \frac{4x}{x^2+1}\right)$$

$$\boxed{g'(x) = \frac{\sin^2 x \tan^4 x}{(x^2+1)^2}\left(\frac{2\cos x}{\sin x} + \frac{4\sec^2 x}{\tan x} - \frac{4x}{x^2+1}\right)}$$

6. Evaluate the following integrals.

4 (a) $\int_0^1 2^{-4x}\,dx$

LET $u = -4x$

$du = -4\,dx \rightarrow -\frac{1}{4}\,du = dx$

$$\rightsquigarrow -\frac{1}{4}\int_0^{-4} 2^u\,du = -\frac{1}{4\ln 2}\,2^u \Big|_0^{-4}$$

$$\boxed{= -\frac{1}{4\ln 2}\left(2^{-4} - 1\right) = \frac{15}{64\ln 2}}$$

4 (b) $\int \frac{2-x^2}{6x-x^3}\,dx$

LET $u = 6x - x^3$

$du = (6 - 3x^2)\,dx$

$\frac{1}{3}\,du = (2 - x^2)\,dx$

$$\rightsquigarrow \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C$$

$$\rightsquigarrow \boxed{\frac{1}{3}\ln|6x - x^3| + C}$$

7. The half-life of cesium-137 is 30 years. Suppose we have a 100-mg sample.

4 (a) Find the mass that remains after t years.

$m(t) = m_0 e^{kt}$. GIVEN (1) $m_0 = m(0) = 100$

(2) $m(30) = \frac{1}{2} m_0 = 50$

$\therefore\ m(30) = 100 e^{30k} = 50$

$e^{30k} = \frac{1}{2}$

$30k = \ln(\frac{1}{2})$

$k = \frac{1}{30} \ln(\frac{1}{2})$

$$\boxed{\begin{array}{l} \text{So } m(t) = 100 e^{\ln(\frac{1}{2}) \cdot \frac{1}{30} \cdot t} \\ \therefore\ m(t) = 100 (\frac{1}{2})^{t/30} \end{array}}$$

4 (b) After how long will only 1 mg remain?

SOLVE FOR t: $m(t) = 100(\frac{1}{2})^{t/30} = 1$

$(\frac{1}{2})^{t/30} = \frac{1}{100}$

$\frac{t}{30} \ln(\frac{1}{2}) = \ln(\frac{1}{100})$

$$\boxed{t = \frac{30 \ln(\frac{1}{100})}{\ln(\frac{1}{2})}}$$