Extra Credit

MATH 202-ST

Due Tuesday, 2/27/2018

Given any two distinct points $(x_1, y_1), (x_2, y_2)$ in the plane, there is exactly one straight line

$$y = ax + b$$

which passes through both points. More interestingly, given any two distinct points $(x_1, y_1), (x_2, y_2)$ in the plane, there is exactly one *exponential* curve

$$y = ae^{bx}$$

which passes through both points. In other words, for both the straight line and the exponential curve, knowing two points on the graph determines the two constants a and b.

Now consider a parabolic curve

$$y = ax^2 + bx + c. \tag{1}$$

Show that given any three distinct points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ in the plane, there is a parabolic curve which passes through all three points. Write down an equation for such a curve in any form you like. Briefly explain how you know its graph is a parabola, and how you know its graph passes through the three points. Then put that equation in the form of equation (1), clearly stating what the values of a, b, and c are, in terms of the coordinates $x_1, x_2, x_3, y_1, y_2, y_3$.

This extra credit is worth up to 5 bonus points on Quiz 1.

For further thinking (not for extra credit): A polynomial of degree n can be made to pass through how many arbitrary distinct points? How would you write down the equation of such a polynomial?