▪ Figure 8 shows the interpretation of the arc length function in Example 4. Figure 9 shows the graph of this arc length function. Why is $s(x)$ negative when x is less than 1?

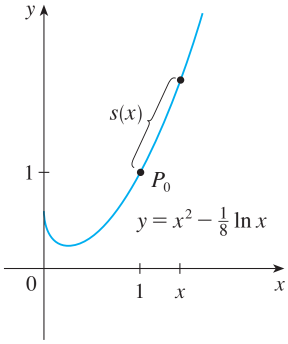


FIGURE 8

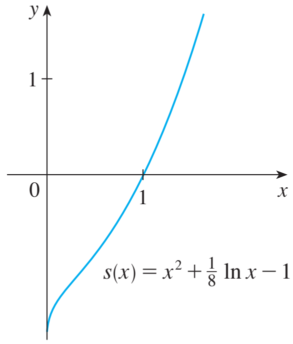


FIGURE 9

7.4 EXERCISES

1. Use the arc length formula (3) to find the length of the curve $y = 2 - 3x$, $-2 \leqslant x \leqslant 1$. Check your answer by noting that the curve is a line segment and calculating its length by the distance formula.

2. Use the arc length formula to find the length of the curve $y = \sqrt{4 - x^2}$, $0 \leqslant x \leqslant 2$. Check your answer by noting that the curve is a quarter-circle.

3–14 ▪ Find the length of the curve.

3. $y = 1 + 6x^{3/2}, \quad 0 \leqslant x \leqslant 1$

4. $y^2 = 4(x + 4)^3, \quad 0 \leqslant x \leqslant 2, \quad y > 0$

5. $y = \dfrac{x^5}{6} + \dfrac{1}{10x^3}, \quad 1 \leqslant x \leqslant 2$

6. $y = \dfrac{x^2}{2} - \dfrac{\ln x}{4}, \quad 2 \leqslant x \leqslant 4$

7. $x = \frac{1}{3}\sqrt{y}\,(y - 3), \quad 1 \leqslant y \leqslant 9$

8. $y = \ln(\cos x), \quad 0 \leqslant x \leqslant \pi/3$

9. $y = \ln(\sec x), \quad 0 \leqslant x \leqslant \pi/4$

10. $y = \ln x, \quad 1 \leqslant x \leqslant \sqrt{3}$

11. $y = \cosh x, \quad 0 \leqslant x \leqslant 1$

12. $y^2 = 4x, \quad 0 \leqslant y \leqslant 2$

13. $y = e^x, \quad 0 \leqslant x \leqslant 1$

14. $y = \ln\left(\dfrac{e^x + 1}{e^x - 1}\right), \quad a \leqslant x \leqslant b, \ a > 0$

15–18 ▪ Set up, but do not evaluate, an integral for the length of the curve.

15. $y = \cos x, \quad 0 \leqslant x \leqslant 2\pi$

16. $y = 2^x, \quad 0 \leqslant x \leqslant 3$

17. $x = y + y^3, \quad 1 \leqslant y \leqslant 4$

18. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

19–22 ▪ Use Simpson's Rule with $n = 10$ to estimate the arc length of the curve. Compare your answer with the value of the integral produced by your calculator.

19. $y = xe^{-x}, \quad 0 \leqslant x \leqslant 5$

20. $x = y + \sqrt{y}, \quad 1 \leqslant y \leqslant 2$

21. $y = \sec x, \quad 0 \leqslant x \leqslant \pi/3$

22. $y = x \ln x, \quad 1 \leqslant x \leqslant 3$

CAS 23. Use either a computer algebra system or a table of integrals to find the *exact* length of the arc of the curve $x = \ln(1 - y^2)$ that lies between the points $(0, 0)$ and $\left(\ln \frac{3}{4}, \frac{1}{2}\right)$.

CAS 24. Use either a computer algebra system or a table of integrals to find the *exact* length of the arc of the curve $y = x^{4/3}$ that lies between the points $(0, 0)$ and $(1, 1)$. If your CAS has trouble evaluating the integral, make a substitution that changes the integral into one that the CAS can evaluate.

25. Sketch the curve with equation $x^{2/3} + y^{2/3} = 1$ and use symmetry to find its length.

26. (a) Sketch the curve $y^3 = x^2$.
(b) Use Formulas 3 and 4 to set up two integrals for the arc length from $(0, 0)$ to $(1, 1)$. Observe that one of these is an improper integral and evaluate both of them.
(c) Find the length of the arc of this curve from $(-1, 1)$ to $(8, 4)$.

27. Find the arc length function for the curve $y = 2x^{3/2}$ with starting point $P_0(1, 2)$.

28. (a) Graph the curve $y = \frac{1}{3}x^3 + 1/(4x)$, $x > 0$.
(b) Find the arc length function for this curve with starting point $P_0\left(1, \frac{7}{12}\right)$.
(c) Graph the arc length function.

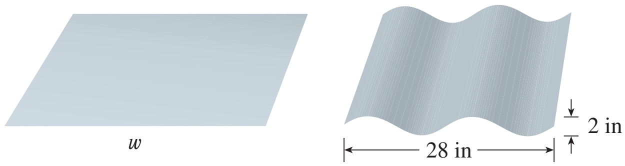
29. A hawk flying at 15 m/s at an altitude of 180 m accidentally drops its prey. The parabolic trajectory of the falling prey is described by the equation

$$y = 180 - \frac{x^2}{45}$$

until it hits the ground, where y is its height above the ground and x is the horizontal distance traveled in meters. Calculate the distance traveled by the prey from the time it is dropped until the time it hits the ground. Express your answer correct to the nearest tenth of a meter.

30. A steady wind blows a kite due west. The kite's height above ground from horizontal position $x = 0$ to $x = 80$ ft is given by $y = 150 - \frac{1}{40}(x - 50)^2$. Find the distance traveled by the kite.

31. A manufacturer of corrugated metal roofing wants to produce panels that are 28 in. wide and 2 in. thick by processing flat sheets of metal as shown in the figure. The profile of the roofing takes the shape of a sine wave. Verify that the sine curve has equation $y = \sin(\pi x/7)$ and find the width w of a flat metal sheet that is needed to make a 28-inch panel. (Use your calculator to evaluate the integral correct to four significant digits.)



32. The curves with equations $x^n + y^n = 1$, $n = 4, 6, 8, \ldots$, are called **fat circles**. Graph the curves with $n = 2, 4, 6, 8$, and 10 to see why. Set up an integral for the length L_{2k} of the fat circle with $n = 2k$. Without attempting to evaluate this integral, state the value of $\lim_{k\to\infty} L_{2k}$.

7.5 APPLICATIONS TO PHYSICS AND ENGINEERING

▪ As a consequence of a calculation of work, you will be able to compute the velocity needed for a rocket to escape the Earth's gravitational field. (See Exercise 22.)

Among the many applications of integral calculus to physics and engineering, we consider three: work, force due to water pressure, and centers of mass. As with our previous applications to geometry (areas, volumes, and lengths), our strategy is to break up the physical quantity into a large number of small parts, approximate each small part, add the results, take the limit, and evaluate the resulting integral.

WORK

The term *work* is used in everyday language to mean the total amount of effort required to perform a task. In physics it has a technical meaning that depends on the idea of a *force*. Intuitively, you can think of a force as describing a push or pull on an object—for example, a horizontal push of a book across a table or the downward pull of the Earth's gravity on a ball. In general, if an object moves along a straight line with position function $s(t)$, then the **force** F on the object (in the same direction) is defined by Newton's Second Law of Motion as the product of its mass m and its acceleration:

$$\boxed{1} \qquad F = m\frac{d^2s}{dt^2}$$

In the SI metric system, the mass is measured in kilograms (kg), the displacement in meters (m), the time in seconds (s), and the force in newtons ($\text{N} = \text{kg}\cdot\text{m/s}^2$). Thus a force of 1 N acting on a mass of 1 kg produces an acceleration of 1 m/s^2. In the US Customary system the fundamental unit is chosen to be the unit of force, which is the pound.

In the case of constant acceleration, the force F is also constant and the work done is defined to be the product of the force F and the distance d that the object moves:

$$\boxed{2} \qquad W = Fd \qquad \text{work} = \text{force} \times \text{distance}$$

If F is measured in newtons and d in meters, then the unit for W is a newton-meter,