Final Exam, 19 December 2016
Math 201

Name: _____ **Instructor:** _____

[**1**]10	[**2**]10	[**3**]10	[**4**]10	[**5**]10	[**6**]7	[**7**]10	[**8**]10	[**9**]10	[**10**]13	**TOTAL**

Please leave these boxes blank!

Instructions: Please read each question carefully, show all work, and check afterwards that you have answered all of each question correctly. **Important: No books, calculators, or notes are allowed. Turn off cell phones, alarms, and anything else that makes noises!** You must show **all** your work to receive credit. Any crossed out work will be disregarded (even if correct). Write **one** clear answer with a coherent derivation for each question. Good luck!

[1] (2.5 pts each) Compute $f'(x)$ for each of the functions below:

(a) $f(x) = \cos(x + \frac{1}{x})$

(b) $f(x) = (x^2 + 3\sin x)(3\sqrt{x} - \frac{1}{x^2})$

(c) $f(x) = \frac{x^2 - 5x^5 + x^7}{x^6}$

(d) $f(x) = \frac{1 + \sec x}{\tan x}$

[2] **(2.5 pts each)** Find each integral:

(a) $\int (4x - 5) \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$

(b) $\int\limits_{0}^{\pi/2} (\cos^3(3x) - \cos(3x)) dx$

(c) $\int \frac{x^4}{\sqrt{2x^5+127}}dx$

(d) $\int \frac{\left(1+\frac{2}{x^2}\right)^5}{x^3}dx$

[3] **(2.5 pts each)** Find the limits, or state that the limit does not exist (you must justify your answer):

(a) $\lim_{x \to 3} \frac{x^2 - 3x}{x^2 + x - 12}$

(b) $\lim_{x \to +\infty} \frac{2x^3 + 3x}{5x^3 - x^2 + 27}$

(c) $\lim_{x \to +\infty} \frac{1}{x^2} \sin(x)$

(d) $\lim_{x \to 0} \frac{\tan x}{x + \sin x}$

[4] (10 pts)

(a) (5 pts) Using the limit definition of the derivative, compute $f'(x)$ if $f(x) = x^2 - 2x$ (no credit will be given for any other method).

(c) (5 pts) Find an equation of the tangent line to the graph $y = x^2 - 2x$ at the point $(1, -1)$.

[5] (10 pts)

(a) (5 pts) Let $F(x) = \int_{x^2}^{\sin(x)} \sqrt{1 - t^3} \, dt$. Find $F'(x)$.

(b) (5 pts) Find the equation of the tangent line to the curve $x^2 + 2xy - y^2 + x = 2$ at the point $(1, 2)$.

[6] **(7 pts)** When a circular plate is heated in an oven, its radius increases at the rate of 0.01 cm/min. At what rate is the plate's area increasing when the radius is 50cm? Be sure to include units in your answer.

[7] (5 pts each)

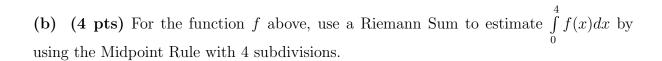**(a)** Find an approximation for $\tan(0.01)$ using calculus.

(b) Suppose $f(x)$ is a differentiable function such that $f(1) = 2$ and $f'(x) \leq 5$ for all x. What is the largest possible value for $f(4)$?

[8] (10 pts)

(a) (2 pts each) Let $f(x) = \begin{cases} x & \text{if } x < 1, \\ 2 & \text{if } x = 1, \\ x^2 & \text{if } x > 1. \end{cases}$

(i) Sketch the graph of $y = f(x)$ for $x \in [0, 4]$.

(ii) Find $\lim_{x \to 1} f(x)$.

(iii) Is $f(x)$ continuous at the point $x = 1$? Please justify your answer.

(b) **(4 pts)** For the function f above, use a Riemann Sum to estimate $\int\limits_{0}^{4} f(x)dx$ by using the Midpoint Rule with 4 subdivisions.

[9] (10 pts) A cylindrical can is made of two different materials: the sides are made of a material that costs 1 dollar per square foot, while the top and bottom are made of a material that costs 2 dollars per square foot.

If the total volume of the can must be 1 cubic feet, find the dimensions of the can that minimize cost.

[10] **(13 pts)** For the function $f(x) = \frac{x^2}{x^2-4}$, you are given (do not compute!) that

$$f'(x) = \frac{-8x}{(x^2-4)^2} \quad \text{and} \quad f''(x) = \frac{8(3x^2+4)}{(x^2-4)^3}.$$

(a) Find the domain of $f(x)$.

(b) Find the coordinates of all intercepts, and the equations of all asymptotes, of the graph of $y = f(x)$.

(c) In what intervals is the function f increasing? decreasing?

(d) In what intervals is the graph of f concave up? concave down?

(e) Find the coordinates of all local maxima, local minima, and points of inflection of the function f.

(f) Sketch the graph of $y = f(x)$. Label the features you found in items b and e.