MATH 195

§4.4 LAWS OF LOGARITHMS

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6/13/2016

#1-5, 9, 11, 13, 16, 17, 19, 32, 35, 37, 43, 45, 46, 51, 55-57

1. SUM

$$\log_5 25 + \log_5 125 = 2 + 3 = 5$$

2. DIFFERENCE

$$\log_5 25 - \log_5 125 = 2 - 3 = -1$$

3. POWER / EXPONENT

$$10 \cdot \log_5 25 = 10 \cdot 2 = 20$$

4. $\log\left(\frac{x^2 y}{z}\right) = \log(x^2 y) - \log(z)$

$$= \boxed{2\log x + \log y - \log z}$$

5. $\log\left(\frac{x^2 y}{z}\right)$ ← NOTE: $\log = \log_{10}$

9. $\log 50 + \log 200 = \log 10000$

$= \boxed{4}$ (BECAUSE $10^4 = 10000$)

11. $\log_2 60 - \log_2 15 = \log_2 \frac{60}{15}$

$= \log_2 4 = \boxed{2}$ (BECAUSE $2^2 = 4$)

13. $\frac{1}{4} \log_3 81 = \log_3 (81^{1/4}) = \log_3 3$

$= \boxed{1}$

16. $\log_5 \frac{1}{\sqrt{125}} = \underbrace{\log_5 1}_{0} - \frac{1}{2} \log_5 125$

$= \boxed{-\frac{3}{2}}$

17. $\log_2 6 - \log_2 15 + \log_2 20$

$= \log_2 \left(\frac{6 \cdot 20}{15} \right) = \log_2 8 = \boxed{3}$

19. $\log_4 16^{100} = 100 \log_4 16 = 100 \cdot 2$

$= \boxed{200}$

32. $\log_3 (x\sqrt{y}) = \boxed{\log_3 x + \frac{1}{2} \log_3 y}$

35. $\log_5\left(\frac{3x^2}{y^3}\right) = \log_5(3x^2) - \log_5(y^3)$

$$= \boxed{\log_5 3 + 2\log_5 x - 3\log_5 y}$$

37. $\log_3\left(\frac{\sqrt{3x^5}}{y}\right) = \log_3\sqrt{3x^5} - \log_3(y)$

$$= \frac{1}{2}\log_3 3x^5 - \log_3 y$$

$$= \frac{1}{2}\left(\log_3 3 + 5\log_3 x\right) - \log_3 y$$

$$= \boxed{\frac{1}{2}\left(1 + 5\log_3 x\right) - \log_3 y}$$

43. $\ln\left(x\sqrt{\frac{y}{z}}\right) = \ln x + \frac{1}{2}\ln\frac{y}{z}$

$$= \boxed{\ln x + \frac{1}{2}\left(\ln y - \ln z\right)}$$

45. $\log\sqrt[4]{x^2+y^2} = \boxed{\frac{1}{4}\log\left(x^2+y^2\right)}$ THAT'S IT!

46. $\log\left(\frac{x}{\sqrt[3]{1-x}}\right) = \boxed{\log x - \frac{1}{3}\log(1-x)}$

51. $2\log x - 3\log(x+1) = \log x^2 - \log\left((x+1)^3\right)$

$= \boxed{\log\left(\frac{x^2}{(x+1)^3}\right)}$

55. $\ln(a+b) + \ln(a-b) - 2\ln c$

$= \ln\frac{(a+b)(a-b)}{c^2} = \boxed{\ln\left(\frac{a^2-b^2}{c^2}\right)}$

56. $2\left(\log_5 x + 2\log_5 y - 3\log_5 z\right)$

$= 2\left(\log\frac{xy^2}{z^3}\right) = \boxed{\log\left(\left(\frac{xy^2}{z^3}\right)^2\right)}$

57. $\frac{1}{3}\log\left((x+2)^3\right) + \frac{1}{2}\left[\log x^4 - \log\left((x^2-x-6)^2\right)\right]$

$= \log\left((x+2)^{3/3}\right) + \frac{1}{2}\log\left(\frac{x^4}{(x^2-x-6)^2}\right)$

$= \log(x+2) + \log\left(\left(\frac{x^4}{(x-3)^2(x+2)^2}\right)^{1/2}\right) = \log\left(\cancel{(x+2)}\cdot\frac{x^2}{(x-3)\cancel{(x+2)}}\right)$

$= \boxed{\log\frac{x^2}{x-3}}$