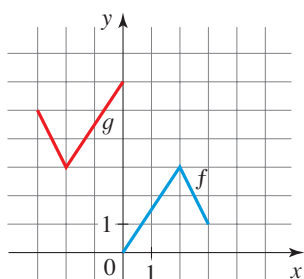


101. The functions f and g are described graphically in the figure.



102. **DISCUSS: Sums of Even and Odd Functions** If f and g are both even functions, is $f + g$ necessarily even? If both are

odd, is their sum necessarily odd? What can you say about the sum if one is odd and one is even? In each case, prove your answer.

103. **DISCUSS: Products of Even and Odd Functions** Answer the same questions as in Exercise 102, except this time consider the product of f and g instead of the sum.

104. **DISCUSS: Even and Odd Power Functions** What must be true about the integer n if the function

$$f(x) = x^n$$

is an even function? If it is an odd function? Why do you think the names “even” and “odd” were chosen for these function properties?

2.7 COMBINING FUNCTIONS

- Sums, Differences, Products, and Quotients
- Composition of Functions
- Applications of Composition

In this section we study different ways to combine functions to make new functions.

■ Sums, Differences, Products, and Quotients

The sum of f and g is defined by

$$(f + g)(x) = f(x) + g(x)$$

The name of the new function is “ $f + g$.” So this $+$ sign stands for the operation of addition of *functions*.

The $+$ sign on the right side, however, stands for addition of the *numbers* $f(x)$ and $g(x)$.

Two functions f and g can be combined to form new functions $f + g$, $f - g$, fg , and f/g in a manner similar to the way we add, subtract, multiply, and divide real numbers. For example, we define the function $f + g$ by

$$(f + g)(x) = f(x) + g(x)$$

The new function $f + g$ is called the **sum** of the functions f and g ; its value at x is $f(x) + g(x)$. Of course, the sum on the right-hand side makes sense only if both $f(x)$ and $g(x)$ are defined, that is, if x belongs to the domain of f and also to the domain of g . So if the domain of f is A and the domain of g is B , then the domain of $f + g$ is the intersection of these domains, that is, $A \cap B$. Similarly, we can define the **difference** $f - g$, the **product** fg , and the **quotient** f/g of the functions f and g . Their domains are $A \cap B$, but in the case of the quotient we must remember not to divide by 0.

ALGEBRA OF FUNCTIONS

Let f and g be functions with domains A and B . Then the functions $f + g$, $f - g$, fg , and f/g are defined as follows.

$$(f + g)(x) = f(x) + g(x) \quad \text{Domain } A \cap B$$

$$(f - g)(x) = f(x) - g(x) \quad \text{Domain } A \cap B$$

$$(fg)(x) = f(x)g(x) \quad \text{Domain } A \cap B$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \text{Domain } \{x \in A \cap B \mid g(x) \neq 0\}$$

EXAMPLE 1 ■ Combinations of Functions and Their Domains

Let $f(x) = \frac{1}{x-2}$ and $g(x) = \sqrt{x}$.

- (a) Find the functions $f + g$, $f - g$, fg , and f/g and their domains.
 (b) Find $(f + g)(4)$, $(f - g)(4)$, $(fg)(4)$, and $(f/g)(4)$.

SOLUTION

- (a) The domain of f is $\{x \mid x \neq 2\}$, and the domain of g is $\{x \mid x \geq 0\}$. The intersection of the domains of f and g is

$$\{x \mid x \geq 0 \text{ and } x \neq 2\} = [0, 2) \cup (2, \infty)$$

Thus we have

$$(f + g)(x) = f(x) + g(x) = \frac{1}{x-2} + \sqrt{x} \quad \text{Domain } \{x \mid x \geq 0 \text{ and } x \neq 2\}$$

$$(f - g)(x) = f(x) - g(x) = \frac{1}{x-2} - \sqrt{x} \quad \text{Domain } \{x \mid x \geq 0 \text{ and } x \neq 2\}$$

$$(fg)(x) = f(x)g(x) = \frac{\sqrt{x}}{x-2} \quad \text{Domain } \{x \mid x \geq 0 \text{ and } x \neq 2\}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{1}{(x-2)\sqrt{x}} \quad \text{Domain } \{x \mid x > 0 \text{ and } x \neq 2\}$$

Note that in the domain of f/g we exclude 0 because $g(0) = 0$.

- (b) Each of these values exist because $x = 4$ is in the domain of each function:

$$(f + g)(4) = f(4) + g(4) = \frac{1}{4-2} + \sqrt{4} = \frac{5}{2}$$

$$(f - g)(4) = f(4) - g(4) = \frac{1}{4-2} - \sqrt{4} = -\frac{3}{2}$$

$$(fg)(4) = f(4)g(4) = \left(\frac{1}{4-2}\right)\sqrt{4} = 1$$

$$\left(\frac{f}{g}\right)(4) = \frac{f(4)}{g(4)} = \frac{1}{(4-2)\sqrt{4}} = \frac{1}{4}$$

To divide fractions, invert the denominator and multiply:

$$\begin{aligned} \frac{1/(x-2)}{\sqrt{x}} &= \frac{1/(x-2)}{\sqrt{x}/1} \\ &= \frac{1}{x-2} \cdot \frac{1}{\sqrt{x}} \\ &= \frac{1}{(x-2)\sqrt{x}} \end{aligned}$$

 Now Try Exercise 9


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DISCOVERY PROJECT**Iteration and Chaos**

The *iterates* of a function f at a point x are the numbers $f(x)$, $f(f(x))$, $f(f(f(x)))$, and so on. We examine iterates of the *logistic function*, which models the population of a species with limited potential for growth (such as lizards on an island or fish in a pond). Iterates of the model can help us to predict whether the population will eventually stabilize or whether it will fluctuate chaotically. You can find the project at **www.stewartmath.com**.

The graph of the function $f + g$ can be obtained from the graphs of f and g by **graphical addition**. This means that we add corresponding y-coordinates, as illustrated in the next example.

EXAMPLE 2 ■ Using Graphical Addition

The graphs of f and g are shown in Figure 1. Use graphical addition to graph the function $f + g$.

SOLUTION We obtain the graph of $f + g$ by “graphically adding” the value of $f(x)$ to $g(x)$ as shown in Figure 2. This is implemented by copying the line segment PQ on top of PR to obtain the point S on the graph of $f + g$.

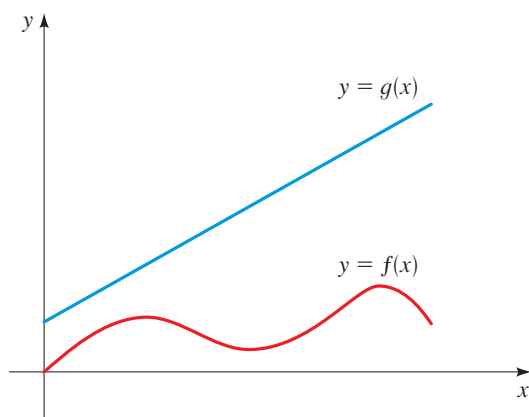


FIGURE 1

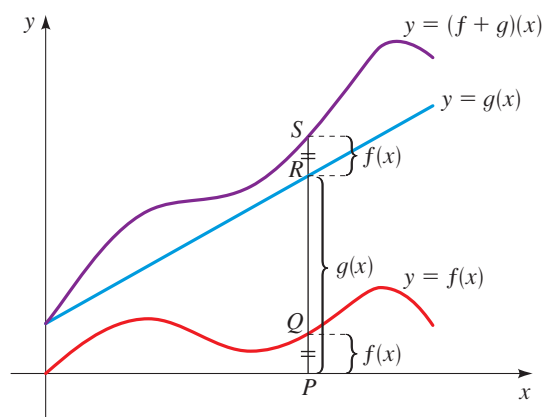


FIGURE 2 Graphical addition

 **Now Try Exercise 21**

■ Composition of Functions

Now let's consider a very important way of combining two functions to get a new function. Suppose $f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$. We may define a new function h as

$$h(x) = f(g(x)) = f(x^2 + 1) = \sqrt{x^2 + 1}$$

The function h is made up of the functions f and g in an interesting way: Given a number x , we first apply the function g to it, then apply f to the result. In this case, f is the rule “take the square root,” g is the rule “square, then add 1,” and h is the rule “square, then add 1, then take the square root.” In other words, we get the rule h by applying the rule g and then the rule f . Figure 3 shows a machine diagram for h .

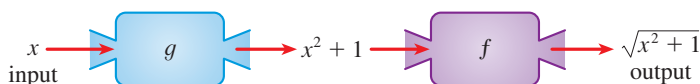


FIGURE 3 The h machine is composed of the g machine (first) and then the f machine.

In general, given any two functions f and g , we start with a number x in the domain of g and find its image $g(x)$. If this number $g(x)$ is in the domain of f , we can then calculate the value of $f(g(x))$. The result is a new function $h(x) = f(g(x))$ that is obtained by substituting g into f . It is called the *composition* (or *composite*) of f and g and is denoted by $f \circ g$ (“ f composed with g ”).

COMPOSITION OF FUNCTIONS

Given two functions f and g , the **composite function** $f \circ g$ (also called the **composition** of f and g) is defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f . In other words, $(f \circ g)(x)$ is defined whenever both $g(x)$ and $f(g(x))$ are defined. We can picture $f \circ g$ using an arrow diagram (Figure 4).

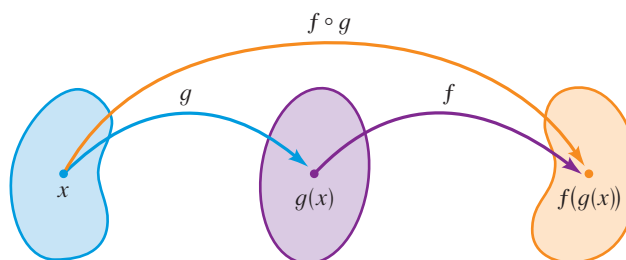


FIGURE 4 Arrow diagram for $f \circ g$

EXAMPLE 3 ■ Finding the Composition of Functions

Let $f(x) = x^2$ and $g(x) = x - 3$.

- (a) Find the functions $f \circ g$ and $g \circ f$ and their domains.
 (b) Find $(f \circ g)(5)$ and $(g \circ f)(7)$.

SOLUTION

- (a) We have

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) && \text{Definition of } f \circ g \\ &= f(x - 3) && \text{Definition of } g \\ &= (x - 3)^2 && \text{Definition of } f \end{aligned}$$

$$\begin{aligned} \text{and} \quad (g \circ f)(x) &= g(f(x)) && \text{Definition of } g \circ f \\ &= g(x^2) && \text{Definition of } f \\ &= x^2 - 3 && \text{Definition of } g \end{aligned}$$

The domains of both $f \circ g$ and $g \circ f$ are $\mathbb{R}$.

- (b) We have

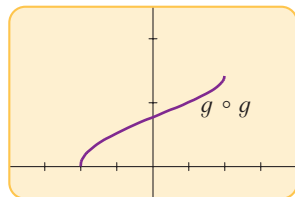
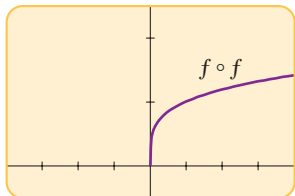
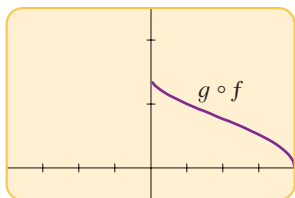
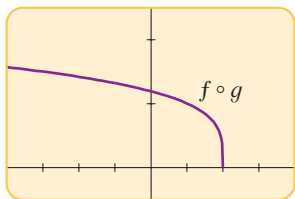
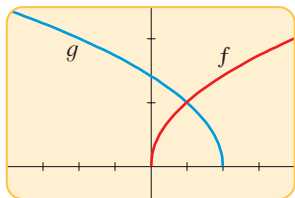
$$\begin{aligned} (f \circ g)(5) &= f(g(5)) = f(2) = 2^2 = 4 \\ (g \circ f)(7) &= g(f(7)) = g(49) = 49 - 3 = 46 \end{aligned}$$

 **Now Try Exercises 27 and 49**

In Example 3, f is the rule “square,” and g is the rule “subtract 3.” The function $f \circ g$ first subtracts 3 and then squares; the function $g \circ f$ first squares and then subtracts 3.

You can see from Example 3 that, in general, $f \circ g \neq g \circ f$. Remember that the notation $f \circ g$ means that the function g is applied first and then f is applied second.

The graphs of f and g of Example 4, as well as those of $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$, are shown below. These graphs indicate that the operation of composition can produce functions that are quite different from the original functions.



EXAMPLE 4 ■ Finding the Composition of Functions

If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$, find the following functions and their domains.

- (a) $f \circ g$ (b) $g \circ f$ (c) $f \circ f$ (d) $g \circ g$

SOLUTION

$$\begin{aligned} \text{(a)} \quad (f \circ g)(x) &= f(g(x)) && \text{Definition of } f \circ g \\ &= f(\sqrt{2-x}) && \text{Definition of } g \\ &= \sqrt{\sqrt{2-x}} && \text{Definition of } f \\ &= \sqrt[4]{2-x} \end{aligned}$$

The domain of $f \circ g$ is $\{x \mid 2-x \geq 0\} = \{x \mid x \leq 2\} = (-\infty, 2]$.

$$\begin{aligned} \text{(b)} \quad (g \circ f)(x) &= g(f(x)) && \text{Definition of } g \circ f \\ &= g(\sqrt{x}) && \text{Definition of } f \\ &= \sqrt{2-\sqrt{x}} && \text{Definition of } g \end{aligned}$$

For $\sqrt{x}$ to be defined, we must have $x \geq 0$. For $\sqrt{2-\sqrt{x}}$ to be defined, we must have $2-\sqrt{x} \geq 0$, that is, $\sqrt{x} \leq 2$, or $x \leq 4$. Thus we have $0 \leq x \leq 4$, so the domain of $g \circ f$ is the closed interval $[0, 4]$.

$$\begin{aligned} \text{(c)} \quad (f \circ f)(x) &= f(f(x)) && \text{Definition of } f \circ f \\ &= f(\sqrt{x}) && \text{Definition of } f \\ &= \sqrt{\sqrt{x}} && \text{Definition of } f \\ &= \sqrt[4]{x} \end{aligned}$$

The domain of $f \circ f$ is $[0, \infty)$.

$$\begin{aligned} \text{(d)} \quad (g \circ g)(x) &= g(g(x)) && \text{Definition of } g \circ g \\ &= g(\sqrt{2-x}) && \text{Definition of } g \\ &= \sqrt{2-\sqrt{2-x}} && \text{Definition of } g \end{aligned}$$

This expression is defined when both $2-x \geq 0$ and $2-\sqrt{2-x} \geq 0$. The first inequality means $x \leq 2$, and the second is equivalent to $\sqrt{2-x} \leq 2$, or $2-x \leq 4$, or $x \geq -2$. Thus $-2 \leq x \leq 2$, so the domain of $g \circ g$ is $[-2, 2]$.

Now Try Exercise 55

It is possible to take the composition of three or more functions. For instance, the composite function $f \circ g \circ h$ is found by first applying h , then g , and then f as follows:

$$(f \circ g \circ h)(x) = f(g(h(x)))$$

EXAMPLE 5 ■ A Composition of Three Functions

Find $f \circ g \circ h$ if $f(x) = x/(x+1)$, $g(x) = x^{10}$, and $h(x) = x+3$.

SOLUTION

$$\begin{aligned} (f \circ g \circ h)(x) &= f(g(h(x))) && \text{Definition of } f \circ g \circ h \\ &= f(g(x+3)) && \text{Definition of } h \\ &= f((x+3)^{10}) && \text{Definition of } g \\ &= \frac{(x+3)^{10}}{(x+3)^{10}+1} && \text{Definition of } f \end{aligned}$$

Now Try Exercise 59

So far, we have used composition to build complicated functions from simpler ones. But in calculus it is useful to be able to “decompose” a complicated function into simpler ones, as shown in the following example.

EXAMPLE 6 ■ Recognizing a Composition of Functions

Given $F(x) = \sqrt[4]{x + 9}$, find functions f and g such that $F = f \circ g$.

SOLUTION Since the formula for F says to first add 9 and then take the fourth root, we let

$$g(x) = x + 9 \quad \text{and} \quad f(x) = \sqrt[4]{x}$$

Then

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) && \text{Definition of } f \circ g \\ &= f(x + 9) && \text{Definition of } g \\ &= \sqrt[4]{x + 9} && \text{Definition of } f \\ &= F(x) \end{aligned}$$

 **Now Try Exercise 63**

Applications of Composition

When working with functions that model real-world situations, we name the variables using letters that suggest the quantity being modeled. We may use t for time, d for distance, V for volume, and so on. For example, if air is being pumped into a balloon, then the radius R of the balloon is a function of the volume V of air pumped into the balloon, say, $R = f(V)$. Also the volume V is a function of the time t that the pump has been working, say, $V = g(t)$. It follows that the radius R is a function of the time t given by $R = f(g(t))$.

EXAMPLE 7 ■ An Application of Composition of Functions

A ship is traveling at 20 mi/h parallel to a straight shoreline. The ship is 5 mi from shore. It passes a lighthouse at noon.

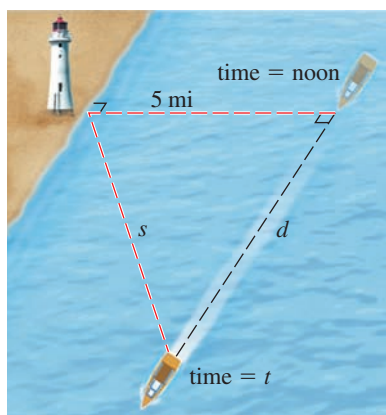


FIGURE 5

distance = rate $\times$ time

- Express the distance s between the lighthouse and the ship as a function of d , the distance the ship has traveled since noon; that is, find f so that $s = f(d)$.
- Express d as a function of t , the time elapsed since noon; that is, find g so that $d = g(t)$.
- Find $f \circ g$. What does this function represent?

SOLUTION We first draw a diagram as in Figure 5.

- We can relate the distances s and d by the Pythagorean Theorem. Thus s can be expressed as a function of d by

$$s = f(d) = \sqrt{25 + d^2}$$

- Since the ship is traveling at 20 mi/h, the distance d it has traveled is a function of t as follows:

$$d = g(t) = 20t$$

- We have

$$\begin{aligned} (f \circ g)(t) &= f(g(t)) && \text{Definition of } f \circ g \\ &= f(20t) && \text{Definition of } g \\ &= \sqrt{25 + (20t)^2} && \text{Definition of } f \end{aligned}$$

The function $f \circ g$ gives the distance of the ship from the lighthouse as a function of time.

 **Now Try Exercise 77**

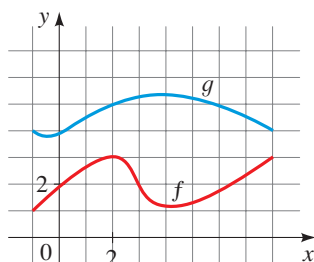
2.7 EXERCISES

CONCEPTS

1. From the graphs of
- f
- and
- g
- in the figure, we find

$$(f + g)(2) = \underline{\hspace{2cm}} \quad (f - g)(2) = \underline{\hspace{2cm}}$$

$$(fg)(2) = \underline{\hspace{2cm}} \quad \left(\frac{f}{g}\right)(2) = \underline{\hspace{2cm}}$$



2. By definition,
- $(f \circ g)(x) = \underline{\hspace{2cm}}$
- . So if
- $g(2) = 5$
- and
- $f(5) = 12$
- , then
- $(f \circ g)(2) = \underline{\hspace{2cm}}$
- .

3. If the rule of the function
- f
- is “add one” and the rule of the function
- g
- is “multiply by 2,” then the rule of
- $f \circ g$
- is

“ $\underline{\hspace{2cm}}$,”and the rule of $g \circ f$ is“ $\underline{\hspace{2cm}}$.”

4. We can express the functions in Exercise 3 algebraically as

$$f(x) = \underline{\hspace{2cm}} \quad g(x) = \underline{\hspace{2cm}}$$

$$(f \circ g)(x) = \underline{\hspace{2cm}} \quad (g \circ f)(x) = \underline{\hspace{2cm}}$$

5–6 ■ Let f and g be functions.

5. (a) The function $(f + g)(x)$ is defined for all values of x that are in the domains of both $\underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$.
- (b) The function $(fg)(x)$ is defined for all values of x that are in the domains of both $\underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$.
- (c) The function $(f/g)(x)$ is defined for all values of x that are in the domains of both $\underline{\hspace{2cm}}$ and $\underline{\hspace{2cm}}$, and $g(x)$ is not equal to $\underline{\hspace{2cm}}$.
6. The composition $(f \circ g)(x)$ is defined for all values of x for which x is in the domain of $\underline{\hspace{2cm}}$ and $g(x)$ is in the domain of $\underline{\hspace{2cm}}$.

SKILLS

- 7–16 ■ Combining Functions Find
- $f + g$
- ,
- $f - g$
- ,
- fg
- , and
- f/g
- and their domains.

7. $f(x) = x$, $g(x) = 2x$

8. $f(x) = x$, $g(x) = \sqrt{x}$

9. $f(x) = x^2 + x$, $g(x) = x^2$

10. $f(x) = 3 - x^2$, $g(x) = x^2 - 4$

11. $f(x) = 5 - x$, $g(x) = x^2 - 3x$

12. $f(x) = x^2 + 2x$, $g(x) = 3x^2 - 1$

13. $f(x) = \sqrt{25 - x^2}$, $g(x) = \sqrt{x + 3}$

14. $f(x) = \sqrt{16 - x^2}$, $g(x) = \sqrt{x^2 - 1}$

15. $f(x) = \frac{2}{x}$, $g(x) = \frac{4}{x + 4}$

16. $f(x) = \frac{2}{x + 1}$, $g(x) = \frac{x}{x + 1}$

- 17–20 ■ Domain Find the domain of the function.

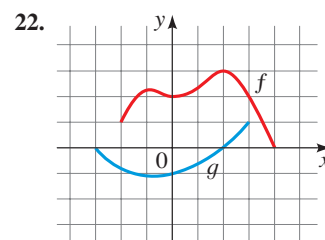
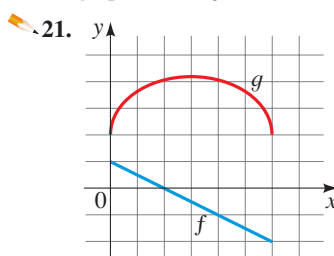
17. $f(x) = \sqrt{x} + \sqrt{3 - x}$

18. $f(x) = \sqrt{x + 4} - \frac{\sqrt{1 - x}}{x}$

19. $h(x) = (x - 3)^{-1/4}$

20. $k(x) = \frac{\sqrt{x + 3}}{x - 1}$

- 21–22 ■ Graphical Addition Use graphical addition to sketch the graph of
- $f + g$
- .



- 23–26 ■ Graphical Addition Draw the graphs of
- f
- ,
- g
- , and
- $f + g$
- on a common screen to illustrate graphical addition.

23. $f(x) = \sqrt{1 + x}$, $g(x) = \sqrt{1 - x}$

24. $f(x) = x^2$, $g(x) = \sqrt{x}$

25. $f(x) = x^2$, $g(x) = \frac{1}{3}x^3$

26. $f(x) = \sqrt[4]{1 - x}$, $g(x) = \sqrt{1 - \frac{x^2}{9}}$

- 27–32 ■ Evaluating Composition of Functions Use
- $f(x) = 2x - 3$
- and
- $g(x) = 4 - x^2$
- to evaluate the expression.

27. (a) $f(g(0))$

(b) $g(f(0))$

28. (a) $f(f(2))$

(b) $g(g(3))$

29. (a) $(f \circ g)(-2)$

(b) $(g \circ f)(-2)$

30. (a) $(f \circ f)(-1)$

(b) $(g \circ g)(-1)$

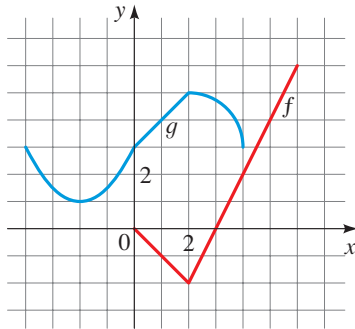
31. (a) $(f \circ g)(x)$

(b) $(g \circ f)(x)$

32. (a) $(f \circ f)(x)$

(b) $(g \circ g)(x)$

33–38 ■ Composition Using a Graph Use the given graphs of f and g to evaluate the expression.



33. $f(g(2))$ 34. $g(f(0))$
 35. $(g \circ f)(4)$ 36. $(f \circ g)(0)$
 37. $(g \circ g)(-2)$ 38. $(f \circ f)(4)$

39–46 ■ Composition Using a Table Use the table to evaluate the expression.

x	1	2	3	4	5	6
$f(x)$	2	3	5	1	6	3
$g(x)$	3	5	6	2	1	4

39. $f(g(2))$ 40. $g(f(2))$
 41. $f(f(1))$ 42. $g(g(2))$
 43. $(f \circ g)(6)$ 44. $(g \circ f)(2)$
 45. $(f \circ f)(5)$ 46. $(g \circ g)(2)$

47–58 ■ Composition of Functions Find the functions $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and their domains.

47. $f(x) = 2x + 3$, $g(x) = 4x - 1$

48. $f(x) = 6x - 5$, $g(x) = \frac{x}{2}$

49. $f(x) = x^2$, $g(x) = x + 1$

50. $f(x) = x^3 + 2$, $g(x) = \sqrt[3]{x}$

51. $f(x) = \frac{1}{x}$, $g(x) = 2x + 4$

52. $f(x) = x^2$, $g(x) = \sqrt{x - 3}$

53. $f(x) = |x|$, $g(x) = 2x + 3$

54. $f(x) = x - 4$, $g(x) = |x + 4|$

55. $f(x) = \frac{x}{x + 1}$, $g(x) = 2x - 1$

56. $f(x) = \frac{1}{\sqrt{x}}$, $g(x) = x^2 - 4x$

57. $f(x) = \frac{x}{x + 1}$, $g(x) = \frac{1}{x}$

58. $f(x) = \frac{2}{x}$, $g(x) = \frac{x}{x + 2}$

59–62 ■ Composition of Three Functions Find $f \circ g \circ h$.

59. $f(x) = x - 1$, $g(x) = \sqrt{x}$, $h(x) = x - 1$

60. $f(x) = \frac{1}{x}$, $g(x) = x^3$, $h(x) = x^2 + 2$

61. $f(x) = x^4 + 1$, $g(x) = x - 5$, $h(x) = \sqrt{x}$

62. $f(x) = \sqrt{x}$, $g(x) = \frac{x}{x - 1}$, $h(x) = \sqrt[3]{x}$

63–68 ■ Expressing a Function as a Composition Express the function in the form $f \circ g$.

63. $F(x) = (x - 9)^5$ 64. $F(x) = \sqrt{x} + 1$

65. $G(x) = \frac{x^2}{x^2 + 4}$ 66. $G(x) = \frac{1}{x + 3}$

67. $H(x) = |1 - x^3|$ 68. $H(x) = \sqrt{1 + \sqrt{x}}$

69–72 ■ Expressing a Function as a Composition Express the function in the form $f \circ g \circ h$.

69. $F(x) = \frac{1}{x^2 + 1}$ 70. $F(x) = \sqrt[3]{\sqrt{x} - 1}$

71. $G(x) = (4 + \sqrt[3]{x})^9$ 72. $G(x) = \frac{2}{(3 + \sqrt{x})^2}$

SKILLS Plus

73. Composing Linear Functions The graphs of the functions

$$f(x) = m_1x + b_1$$

$$g(x) = m_2x + b_2$$

are lines with slopes m_1 and m_2 , respectively. Is the graph of $f \circ g$ a line? If so, what is its slope?

74. Solving an Equation for an Unknown Function Suppose that

$$g(x) = 2x + 1$$

$$h(x) = 4x^2 + 4x + 7$$

Find a function f such that $f \circ g = h$. (Think about what operations you would have to perform on the formula for g to end up with the formula for h .) Now suppose that

$$f(x) = 3x + 5$$

$$h(x) = 3x^2 + 3x + 2$$

Use the same sort of reasoning to find a function g such that $f \circ g = h$.

APPLICATIONS

75–76 ■ Revenue, Cost, and Profit A print shop makes bumper stickers for election campaigns. If x stickers are ordered (where $x < 10,000$), then the price per bumper sticker is $0.15 - 0.000002x$ dollars, and the total cost of producing the order is $0.095x - 0.0000005x^2$ dollars.

75. Use the fact that

$$\text{revenue} = \text{price per item} \times \text{number of items sold}$$

to express $R(x)$, the revenue from an order of x stickers, as a product of two functions of x .

76. Use the fact that

$$\text{profit} = \text{revenue} - \text{cost}$$

to express $P(x)$, the profit on an order of x stickers, as a difference of two functions of x .

-  77. **Area of a Ripple** A stone is dropped in a lake, creating a circular ripple that travels outward at a speed of 60 cm/s.
- Find a function g that models the radius as a function of time.
 - Find a function f that models the area of the circle as a function of the radius.
 - Find $f \circ g$. What does this function represent?



78. **Inflating a Balloon** A spherical balloon is being inflated. The radius of the balloon is increasing at the rate of 1 cm/s.
- Find a function f that models the radius as a function of time.
 - Find a function g that models the volume as a function of the radius.
 - Find $g \circ f$. What does this function represent?

79. **Area of a Balloon** A spherical weather balloon is being inflated. The radius of the balloon is increasing at the rate of 2 cm/s. Express the surface area of the balloon as a function of time t (in seconds).

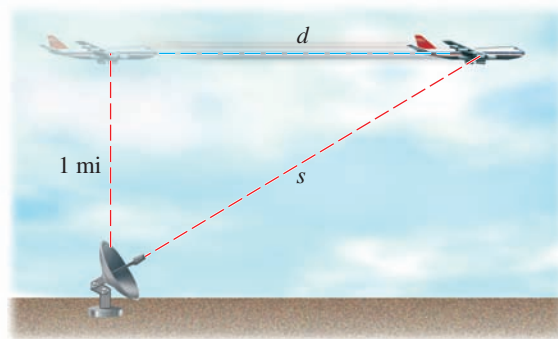
80. **Multiple Discounts** You have a \$50 coupon from the manufacturer that is good for the purchase of a cell phone. The store where you are purchasing your cell phone is offering a 20% discount on all cell phones. Let x represent the regular price of the cell phone.
- Suppose only the 20% discount applies. Find a function f that models the purchase price of the cell phone as a function of the regular price x .
 - Suppose only the \$50 coupon applies. Find a function g that models the purchase price of the cell phone as a function of the sticker price x .
 - If you can use the coupon and the discount, then the purchase price is either $(f \circ g)(x)$ or $(g \circ f)(x)$, depending on the order in which they are applied to the price. Find both $(f \circ g)(x)$ and $(g \circ f)(x)$. Which composition gives the lower price?

81. **Multiple Discounts** An appliance dealer advertises a 10% discount on all his washing machines. In addition, the manufacturer offers a \$100 rebate on the purchase of a

washing machine. Let x represent the sticker price of the washing machine.

- Suppose only the 10% discount applies. Find a function f that models the purchase price of the washer as a function of the sticker price x .
- Suppose only the \$100 rebate applies. Find a function g that models the purchase price of the washer as a function of the sticker price x .
- Find $f \circ g$ and $g \circ f$. What do these functions represent? Which is the better deal?

82. **Airplane Trajectory** An airplane is flying at a speed of 350 mi/h at an altitude of one mile. The plane passes directly above a radar station at time $t = 0$.
- Express the distance s (in miles) between the plane and the radar station as a function of the horizontal distance d (in miles) that the plane has flown.
 - Express d as a function of the time t (in hours) that the plane has flown.
 - Use composition to express s as a function of t .



DISCUSS ■ DISCOVER ■ PROVE ■ WRITE

83. **DISCOVER: Compound Interest** A savings account earns 5% interest compounded annually. If you invest x dollars in such an account, then the amount $A(x)$ of the investment after one year is the initial investment plus 5%; that is,

$$A(x) = x + 0.05x = 1.05x$$

Find

$$A \circ A$$

$$A \circ A \circ A$$

$$A \circ A \circ A \circ A$$

What do these compositions represent? Find a formula for what you get when you compose n copies of A .

84. **DISCUSS: Compositions of Odd and Even Functions** Suppose that

$$h = f \circ g$$

If g is an even function, is h necessarily even? If g is odd, is h odd? What if g is odd and f is odd? What if g is odd and f is even?