

2.1 FUNCTIONS

- Functions All Around Us
- Definition of Function
- Evaluating a Function
- The Domain of a Function
- Four Ways to Represent a Function

In this section we explore the idea of a function and then give the mathematical definition of function.

■ Functions All Around Us

In nearly every physical phenomenon we observe that one quantity depends on another. For example, your height depends on your age, the temperature depends on the date, the cost of mailing a package depends on its weight (see Figure 1). We use the term *function* to describe this dependence of one quantity on another. That is, we say the following:

- Height is a function of age.
- Temperature is a function of date.
- Cost of mailing a package is a function of weight.

The U.S. Post Office uses a simple rule to determine the cost of mailing a first-class parcel on the basis of its weight. But it’s not so easy to describe the rule that relates height to age or the rule that relates temperature to date.

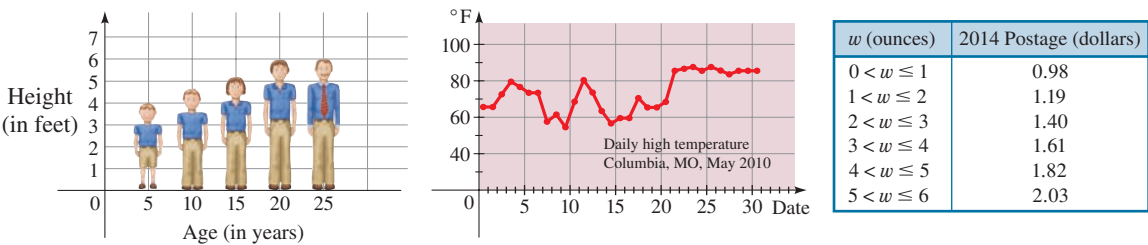


FIGURE 1 Height is a function of age. Temperature is a function of date. Postage is a function of weight.

Can you think of other functions? Here are some more examples:

- The area of a circle is a function of its radius.
- The number of bacteria in a culture is a function of time.
- The weight of an astronaut is a function of her elevation.
- The price of a commodity is a function of the demand for that commodity.

The rule that describes how the area A of a circle depends on its radius r is given by the formula $A = \pi r^2$. Even when a precise rule or formula describing a function is not available, we can still describe the function by a graph. For example, when you turn on a hot water faucet, the temperature of the water depends on how long the water has been running. So we can say:

- The temperature of water from the faucet is a function of time.

Figure 2 shows a rough graph of the temperature T of the water as a function of the time t that has elapsed since the faucet was turned on. The graph shows that the initial temperature of the water is close to room temperature. When the water from the hot water tank reaches the faucet, the water’s temperature T increases quickly. In the next phase,



T is constant at the temperature of the water in the tank. When the tank is drained, T decreases to the temperature of the cold water supply.

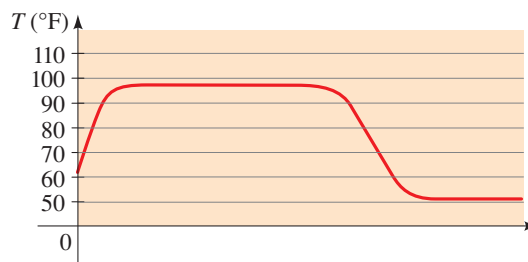


FIGURE 2 Graph of water temperature T as a function of time t

We have previously used letters to stand for numbers. Here we do something quite different: We use letters to represent *rules*.

■ Definition of Function

A function is a rule. To talk about a function, we need to give it a name. We will use letters such as f , g , h , . . . to represent functions. For example, we can use the letter f to represent a rule as follows:

“ f ” is the rule “square the number”

When we write $f(2)$, we mean “apply the rule f to the number 2.” Applying the rule gives $f(2) = 2^2 = 4$. Similarly, $f(3) = 3^2 = 9$, $f(4) = 4^2 = 16$, and in general $f(x) = x^2$.

DEFINITION OF A FUNCTION

A **function** f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B .

The $\sqrt{}$ key on your calculator is a good example of a function as a machine. First you input x into the display. Then you press the key labeled $\sqrt{}$. (On most *graphing* calculators the order of these operations is reversed.) If $x < 0$, then x is not in the domain of this function; that is, x is not an acceptable input, and the calculator will indicate an error. If $x \geq 0$, then an approximation to $\sqrt{x}$ appears in the display, correct to a certain number of decimal places. (Thus the $\sqrt{}$ key on your calculator is not quite the same as the exact mathematical function f defined by $f(x) = \sqrt{x}$.)

We usually consider functions for which the sets A and B are sets of real numbers. The symbol $f(x)$ is read “ f of x ” or “ f at x ” and is called the **value of f at x** , or the **image of x under f** . The set A is called the **domain** of the function. The **range** of f is the set of all possible values of $f(x)$ as x varies throughout the domain, that is,

$$\text{range of } f = \{f(x) \mid x \in A\}$$

The symbol that represents an arbitrary number in the domain of a function f is called an **independent variable**. The symbol that represents a number in the range of f is called a **dependent variable**. So if we write $y = f(x)$, then x is the independent variable and y is the dependent variable.

It is helpful to think of a function as a **machine** (see Figure 3). If x is in the domain of the function f , then when x enters the machine, it is accepted as an **input** and the machine produces an **output** $f(x)$ according to the rule of the function. Thus we can think of the domain as the set of all possible inputs and the range as the set of all possible outputs.



FIGURE 3 Machine diagram of f

Another way to picture a function f is by an **arrow diagram** as in Figure 4(a). Each arrow associates an input from A to the corresponding output in B . Since a function

associates *exactly* one output to each input, the diagram in Figure 4(a) represents a function but the diagram in Figure 4(b) does *not* represent a function.

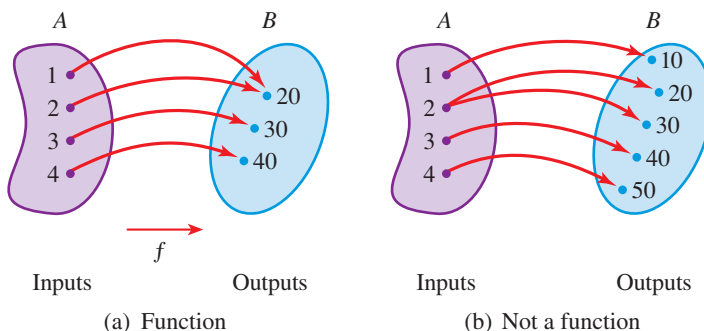


FIGURE 4 Arrow diagrams

EXAMPLE 1 ■ Analyzing a Function

A function f is defined by the formula

$$f(x) = x^2 + 4$$

- Express in words how f acts on the input x to produce the output $f(x)$.
- Evaluate $f(3)$, $f(-2)$, and $f(\sqrt{5})$.
- Find the domain and range of f .
- Draw a machine diagram for f .

SOLUTION

- The formula tells us that f first squares the input x and then adds 4 to the result. So f is the function

“square, then add 4”

- The values of f are found by substituting for x in the formula $f(x) = x^2 + 4$.

$$\begin{aligned} f(3) &= 3^2 + 4 = 13 && \text{Replace } x \text{ by } 3 \\ f(-2) &= (-2)^2 + 4 = 8 && \text{Replace } x \text{ by } -2 \\ f(\sqrt{5}) &= (\sqrt{5})^2 + 4 = 9 && \text{Replace } x \text{ by } \sqrt{5} \end{aligned}$$

- The domain of f consists of all possible inputs for f . Since we can evaluate the formula $f(x) = x^2 + 4$ for every real number x , the domain of f is the set $\mathbb{R}$ of all real numbers.

The range of f consists of all possible outputs of f . Because $x^2 \geq 0$ for all real numbers x , we have $x^2 + 4 \geq 4$, so for every output of f we have $f(x) \geq 4$. Thus the range of f is $\{y \mid y \geq 4\} = [4, \infty)$.

- A machine diagram for f is shown in Figure 5.

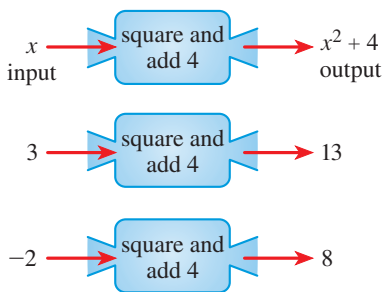


FIGURE 5 Machine diagram

Now Try Exercises 11, 15, 19, and 51

■ Evaluating a Function

In the definition of a function the independent variable x plays the role of a placeholder. For example, the function $f(x) = 3x^2 + x - 5$ can be thought of as

$$f(\text{placeholder}) = 3 \cdot \text{placeholder}^2 + \text{placeholder} - 5$$

To evaluate f at a number, we substitute the number for the placeholder.

EXAMPLE 2 ■ Evaluating a Function

Let $f(x) = 3x^2 + x - 5$. Evaluate each function value.

- (a) $f(-2)$ (b) $f(0)$ (c) $f(4)$ (d) $f(\frac{1}{2})$

SOLUTION To evaluate f at a number, we substitute the number for x in the definition of f .

$$(a) \quad f(-2) = 3 \cdot (-2)^2 + (-2) - 5 = 5$$

$$(b) \quad f(0) = 3 \cdot 0^2 + 0 - 5 = -5$$

$$(c) \quad f(4) = 3 \cdot (4)^2 + 4 - 5 = 47$$

$$(d) \quad f(\frac{1}{2}) = 3 \cdot (\frac{1}{2})^2 + \frac{1}{2} - 5 = -\frac{15}{4}$$

 **Now Try Exercise 21**



A **piecewise defined function** is defined by different formulas on different parts of its domain. The function C of Example 3 is piecewise defined.

EXAMPLE 3 ■ A Piecewise Defined Function

A cell phone plan costs \$39 a month. The plan includes 2 gigabytes (GB) of free data and charges \$15 per gigabyte for any additional data used. The monthly charges are a function of the number of gigabytes of data used, given by

$$C(x) = \begin{cases} 39 & \text{if } 0 \leq x \leq 2 \\ 39 + 15(x - 2) & \text{if } x > 2 \end{cases}$$

Find $C(0.5)$, $C(2)$, and $C(4)$.

SOLUTION Remember that a function is a rule. Here is how we apply the rule for this function. First we look at the value of the input, x . If $0 \leq x \leq 2$, then the value of $C(x)$ is 39. On the other hand, if $x > 2$, then the value of $C(x)$ is $39 + 15(x - 2)$.

Since $0.5 \leq 2$, we have $C(0.5) = 39$.

Since $2 \leq 2$, we have $C(2) = 39$.

Since $4 > 2$, we have $C(4) = 39 + 15(4 - 2) = 69$.

Thus the plan charges \$39 for 0.5 GB, \$39 for 2 GB, and \$69 for 4 GB.

 **Now Try Exercises 31 and 85**

From Examples 2 and 3 we see that the values of a function can change from one input to another. The **net change** in the value of a function f as the input changes from a to b (where $a \leq b$) is given by

$$f(b) - f(a)$$

The next example illustrates this concept.

EXAMPLE 4 ■ Finding Net Change

Let $f(x) = x^2$. Find the net change in the value of f between the given inputs.

- (a) From 1 to 3 (b) From -2 to 2

SOLUTION

(a) The net change is $f(3) - f(1) = 9 - 1 = 8$.

(b) The net change is $f(2) - f(-2) = 4 - 4 = 0$.

 **Now Try Exercise 39**

The values of the function in Example 4 decrease and then increase between -2 and 2, but the net change from -2 to 2 is 0 because $f(-2)$ and $f(2)$ have the same value.

Expressions like the one in part (d) of Example 5 occur frequently in calculus; they are called *difference quotients*, and they represent the average change in the value of f between $x = a$ and $x = a + h$.

EXAMPLE 5 ■ Evaluating a Function

If $f(x) = 2x^2 + 3x - 1$, evaluate the following.

(a) $f(a)$ (b) $f(-a)$ (c) $f(a + h)$ (d) $\frac{f(a + h) - f(a)}{h}, \quad h \neq 0$

SOLUTION

(a) $f(a) = 2a^2 + 3a - 1$
(b) $f(-a) = 2(-a)^2 + 3(-a) - 1 = 2a^2 - 3a - 1$
(c) $f(a + h) = 2(a + h)^2 + 3(a + h) - 1$
 $\quad = 2(a^2 + 2ah + h^2) + 3(a + h) - 1$
 $\quad = 2a^2 + 4ah + 2h^2 + 3a + 3h - 1$
(d) Using the results from parts (c) and (a), we have

$$\begin{aligned} \frac{f(a + h) - f(a)}{h} &= \frac{(2a^2 + 4ah + 2h^2 + 3a + 3h - 1) - (2a^2 + 3a - 1)}{h} \\ &= \frac{4ah + 2h^2 + 3h}{h} = 4a + 2h + 3 \end{aligned}$$

 Now Try Exercise 43

A **table of values** for a function is a table with two headings, one for inputs and one for the corresponding outputs. A table of values helps us to analyze a function numerically, as in the next example.

EXAMPLE 6 ■ The Weight of an Astronaut

If an astronaut weighs 130 lb on the surface of the earth, then her weight when she is h miles above the earth is given by the function

$$w(h) = 130 \left(\frac{3960}{3960 + h} \right)^2$$

- (a) What is her weight when she is 100 mi above the earth?
(b) Construct a table of values for the function w that gives her weight at heights from 0 to 500 mi. What do you conclude from the table?
(c) Find the net change in the astronaut's weight from ground level to a height of 500 mi.

SOLUTION

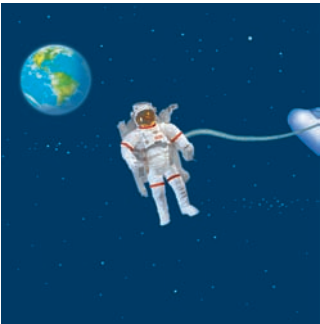
(a) We want the value of the function w when $h = 100$; that is, we must calculate $w(100)$:

$$w(100) = 130 \left(\frac{3960}{3960 + 100} \right)^2 \approx 123.67$$

So at a height of 100 mi she weighs about 124 lb.

(b) The table gives the astronaut's weight, rounded to the nearest pound, at 100-mi increments. The values in the table are calculated as in part (a).

h	$w(h)$
0	130
100	124
200	118
300	112
400	107
500	102



The weight of an object on or near the earth is the gravitational force that the earth exerts on it. When in orbit around the earth, an astronaut experiences the sensation of “weightlessness” because the centripetal force that keeps her in orbit is exactly the same as the gravitational pull of the earth.

The table indicates that the higher the astronaut travels, the less she weighs.

- (c) The net change in the astronaut's weight from $h = 0$ to $h = 500$ is

$$w(500) - w(0) = 102 - 130 = -28$$

The negative sign indicates that the astronaut's weight *decreased* by about 28 lb.

 **Now Try Exercise 79**

■ The Domain of a Function

Recall that the *domain* of a function is the set of all inputs for the function. The domain of a function may be stated explicitly. For example, if we write

$$f(x) = x^2 \quad 0 \leq x \leq 5$$

then the domain is the set of all real numbers x for which $0 \leq x \leq 5$. If the function is given by an algebraic expression and the domain is not stated explicitly, then by convention *the domain of the function is the domain of the algebraic expression—that is, the set of all real numbers for which the expression is defined as a real number*. For example, consider the functions

$$f(x) = \frac{1}{x-4} \quad g(x) = \sqrt{x}$$

The function f is not defined at $x = 4$, so its domain is $\{x \mid x \neq 4\}$. The function g is not defined for negative x , so its domain is $\{x \mid x \geq 0\}$.

EXAMPLE 7 ■ Finding Domains of Functions

Find the domain of each function.

$$(a) f(x) = \frac{1}{x^2 - x} \quad (b) g(x) = \sqrt{9 - x^2} \quad (c) h(t) = \frac{t}{\sqrt{t+1}}$$

SOLUTION

- (a) A rational expression is not defined when the denominator is 0. Since

$$f(x) = \frac{1}{x^2 - x} = \frac{1}{x(x-1)}$$

we see that $f(x)$ is not defined when $x = 0$ or $x = 1$. Thus the domain of f is

$$\{x \mid x \neq 0, x \neq 1\}$$

The domain may also be written in interval notation as

$$(-\infty, 0) \cup (0, 1) \cup (1, \infty)$$

- (b) We can't take the square root of a negative number, so we must have $9 - x^2 \geq 0$. Using the methods of Section 1.8, we can solve this inequality to find that $-3 \leq x \leq 3$. Thus the domain of g is

$$\{x \mid -3 \leq x \leq 3\} = [-3, 3]$$

- (c) We can't take the square root of a negative number, and we can't divide by 0, so we must have $t + 1 > 0$, that is, $t > -1$. So the domain of h is

$$\{t \mid t > -1\} = (-1, \infty)$$

 **Now Try Exercises 55, 59, and 69**

Domains of algebraic expressions are discussed on page 36.

■ Four Ways to Represent a Function

To help us understand what a function is, we have used machine and arrow diagrams. We can describe a specific function in the following four ways:

- verbally (by a description in words)
- algebraically (by an explicit formula)
- visually (by a graph)
- numerically (by a table of values)

A single function may be represented in all four ways, and it is often useful to go from one representation to another to gain insight into the function. However, certain functions are described more naturally by one method than by the others. An example of a verbal description is the following rule for converting between temperature scales:

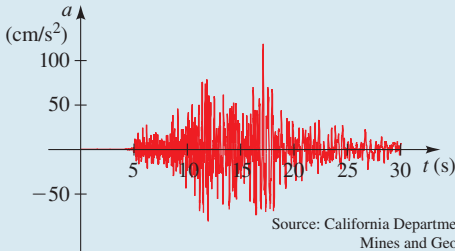
“To find the Fahrenheit equivalent of a Celsius temperature, multiply the Celsius temperature by $\frac{9}{5}$, then add 32.”

In Example 8 we see how to describe this verbal rule or function algebraically, graphically, and numerically. A useful representation of the area of a circle as a function of its radius is the algebraic formula

$$A(r) = \pi r^2$$

The graph produced by a seismograph (see the box below) is a visual representation of the vertical acceleration function $a(t)$ of the ground during an earthquake. As a final example, consider the function $C(w)$, which is described verbally as “the cost of mailing a large first-class letter with weight w .” The most convenient way of describing this function is numerically—that is, using a table of values.

We will be using all four representations of functions throughout this book. We summarize them in the following box.

FOUR WAYS TO REPRESENT A FUNCTION Verbal Using words: “To convert from Celsius to Fahrenheit, multiply the Celsius temperature by $\frac{9}{5}$, then add 32.” Relation between Celsius and Fahrenheit temperature scales	Algebraic Using a formula: $A(r) = \pi r^2$ Area of a circle														
Visual Using a graph:  Source: California Department of Mines and Geology Vertical acceleration during an earthquake	Numerical Using a table of values: <table> <tr> <th>w (ounces)</th><th>$C(w)$ (dollars)</th></tr> <tr> <td>$0 < w \leq 1$</td><td>\$0.98</td></tr> <tr> <td>$1 < w \leq 2$</td><td>\$1.19</td></tr> <tr> <td>$2 < w \leq 3$</td><td>\$1.40</td></tr> <tr> <td>$3 < w \leq 4$</td><td>\$1.61</td></tr> <tr> <td>$4 < w \leq 5$</td><td>\$1.82</td></tr> <tr> <td>$\vdots$</td><td>$\vdots$</td></tr> </table> Cost of mailing a large first-class envelope	w (ounces)	$C(w)$ (dollars)	$0 < w \leq 1$	\$0.98	$1 < w \leq 2$	\$1.19	$2 < w \leq 3$	\$1.40	$3 < w \leq 4$	\$1.61	$4 < w \leq 5$	\$1.82	$\vdots$	$\vdots$
w (ounces)	$C(w)$ (dollars)														
$0 < w \leq 1$	\$0.98														
$1 < w \leq 2$	\$1.19														
$2 < w \leq 3$	\$1.40														
$3 < w \leq 4$	\$1.61														
$4 < w \leq 5$	\$1.82														
$\vdots$	$\vdots$														

EXAMPLE 8 ■ Representing a Function Verbally, Algebraically, Numerically, and Graphically

Let $F(C)$ be the Fahrenheit temperature corresponding to the Celsius temperature C . (Thus F is the function that converts Celsius inputs to Fahrenheit outputs.) The box on page 154 gives a verbal description of this function. Find ways to represent this function

- (a) Algebraically (using a formula)
- (b) Numerically (using a table of values)
- (c) Visually (using a graph)

SOLUTION

- (a) The verbal description tells us that we should first multiply the input C by $\frac{9}{5}$ and then add 32 to the result. So we get

$$F(C) = \frac{9}{5}C + 32$$

- (b) We use the algebraic formula for F that we found in part (a) to construct a table of values:

C (Celsius)	F (Fahrenheit)
-10	14
0	32
10	50
20	68
30	86
40	104

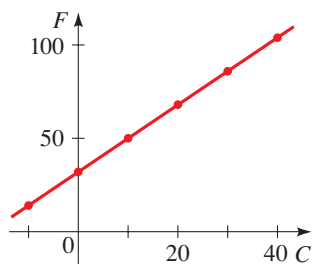


FIGURE 6 Celsius and Fahrenheit

- (c) We use the points tabulated in part (b) to help us draw the graph of this function in Figure 6.



Now Try Exercise 73

2.1 EXERCISES**CONCEPTS**

- If $f(x) = x^3 + 1$, then
 - (a) the value of f at $x = -1$ is $f(\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$.
 - (b) the value of f at $x = 2$ is $f(\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$.
 - (c) the net change in the value of f between $x = -1$ and $x = 2$ is $f(\underline{\hspace{1cm}}) - f(\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$.
- For a function f , the set of all possible inputs is called the of f , and the set of all possible outputs is called the of f .
- (a) Which of the following functions have 5 in their domain?

$$f(x) = x^2 - 3x \qquad g(x) = \frac{x-5}{x} \qquad h(x) = \sqrt{x-10}$$

- (b) For the functions from part (a) that *do* have 5 in their domain, find the value of the function at 5.

4. A function is given algebraically by the formula $f(x) = (x-4)^2 + 3$. Complete these other ways to represent f :

- (a) *Verbal*: “Subtract 4, then and .”
- (b) *Numerical*:

x	$f(x)$
0	19
2	
4	
6	

5. A function f is a rule that assigns to each element x in a set A exactly _____ element(s) called $f(x)$ in a set B . Which of the following tables defines y as a function of x ?

(i) (ii)

x	y
1	5
2	7
3	6
4	8

x	y
1	5
1	7
2	6
3	8

6. Yes or No? If No, give a reason. Let f be a function.

- (a) Is it possible that $f(1) = 5$ and $f(2) = 5$?
 (b) Is it possible that $f(1) = 5$ and $f(1) = 6$?

SKILLS

7–10 ■ Function Notation Express the rule in function notation. (For example, the rule “square, then subtract 5” is expressed as the function $f(x) = x^2 - 5$.)

7. Multiply by 3, then subtract 5

8. Square, then add 2

9. Subtract 1, then square

10. Add 1, take the square root, then divide by 6

11–14 ■ Functions in Words Express the function (or rule) in words.

11. $f(x) = 2x + 3$ 12. $g(x) = \frac{x+2}{3}$
 13. $h(x) = 5(x+1)$ 14. $k(x) = \frac{x^2-4}{3}$

15–16 ■ Machine Diagram Draw a machine diagram for the function.

15. $f(x) = \sqrt{x-1}$ 16. $f(x) = \frac{3}{x-2}$

17–18 ■ Table of Values Complete the table.

17. $f(x) = 2(x-1)^2$ 18. $g(x) = |2x+3|$

x	$f(x)$
-1	
0	
1	
2	
3	

x	$g(x)$
-3	
-2	
0	
1	
3	

19–30 ■ Evaluating Functions Evaluate the function at the indicated values.

19. $f(x) = x^2 - 6$; $f(-3), f(3), f(0), f(\frac{1}{2})$
 20. $f(x) = x^3 + 2x$; $f(-2), f(-1), f(0), f(\frac{1}{2})$

21. $f(x) = \frac{1-2x}{3}$;
 $f(2), f(-2), f(\frac{1}{2}), f(a), f(-a), f(a-1)$

22. $h(x) = \frac{x^2+4}{5}$;
 $h(2), h(-2), h(a), h(-x), h(a-2), h(\sqrt{x})$

23. $f(x) = x^2 + 2x$;
 $f(0), f(3), f(-3), f(a), f(-x), f(\frac{1}{a})$

24. $h(t) = t + \frac{1}{t}$;
 $h(-1), h(2), h(\frac{1}{2}), h(x-1), h(\frac{1}{x})$

25. $g(x) = \frac{1-x}{1+x}$;
 $g(2), g(-1), g(\frac{1}{2}), g(a), g(a-1), g(x^2-1)$

26. $g(t) = \frac{t+2}{t-2}$;
 $g(-2), g(2), g(0), g(a), g(a^2-2), g(a+1)$
 27. $k(x) = -x^2 - 2x + 3$;
 $k(0), k(2), k(-2), k(\sqrt{2}), k(a+2), k(-x), k(x^2)$

28. $k(x) = 2x^3 - 3x^2$;
 $k(0), k(3), k(-3), k(\frac{1}{2}), k(\frac{a}{2}), k(-x), k(x^3)$

29. $f(x) = 2|x-1|$;
 $f(-2), f(0), f(\frac{1}{2}), f(2), f(x+1), f(x^2+2)$

30. $f(x) = \frac{|x|}{x}$;
 $f(-2), f(-1), f(0), f(5), f(x^2), f(\frac{1}{x})$

31–34 ■ Piecewise Defined Functions Evaluate the piecewise defined function at the indicated values.

31. $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x+1 & \text{if } x \geq 0 \end{cases}$
 $f(-2), f(-1), f(0), f(1), f(2)$

32. $f(x) = \begin{cases} 5 & \text{if } x \leq 2 \\ 2x-3 & \text{if } x > 2 \end{cases}$
 $f(-3), f(0), f(2), f(3), f(5)$

33. $f(x) = \begin{cases} x^2+2x & \text{if } x \leq -1 \\ x & \text{if } -1 < x \leq 1 \\ -1 & \text{if } x > 1 \end{cases}$
 $f(-4), f(-\frac{3}{2}), f(-1), f(0), f(25)$

34. $f(x) = \begin{cases} 3x & \text{if } x < 0 \\ x+1 & \text{if } 0 \leq x \leq 2 \\ (x-2)^2 & \text{if } x > 2 \end{cases}$
 $f(-5), f(0), f(1), f(2), f(5)$

35–38 ■ Evaluating Functions Use the function to evaluate the indicated expressions and simplify.

35. $f(x) = x^2 + 1$; $f(x + 2)$, $f(x) + f(2)$

36. $f(x) = 3x - 1$; $f(2x)$, $2f(x)$

37. $f(x) = x + 4$; $f(x^2)$, $(f(x))^2$

38. $f(x) = 6x - 18$; $f\left(\frac{x}{3}\right)$, $\frac{f(x)}{3}$

39–42 ■ Net Change Find the net change in the value of the function between the given inputs.

39. $f(x) = 3x - 2$; from 1 to 5

40. $f(x) = 4 - 5x$; from 3 to 5

41. $g(t) = 1 - t^2$; from -2 to 5

42. $h(t) = t^2 + 5$; from -3 to 6

43–50 ■ Difference Quotient Find $f(a)$, $f(a + h)$, and the difference quotient $\frac{f(a + h) - f(a)}{h}$, where $h \neq 0$.

43. $f(x) = 5 - 2x$

44. $f(x) = 3x^2 + 2$

45. $f(x) = 5$

46. $f(x) = \frac{1}{x + 1}$

47. $f(x) = \frac{x}{x + 1}$

48. $f(x) = \frac{2x}{x - 1}$

49. $f(x) = 3 - 5x + 4x^2$

50. $f(x) = x^3$

51–54 ■ Domain and Range Find the domain and range of the function.

51. $f(x) = 3x$

52. $f(x) = 5x^2 + 4$

53. $f(x) = 3x$, $-2 \leq x \leq 6$

54. $f(x) = 5x^2 + 4$, $0 \leq x \leq 2$

55–72 ■ Domain Find the domain of the function.

55. $f(x) = \frac{1}{x - 3}$

56. $f(x) = \frac{1}{3x - 6}$

57. $f(x) = \frac{x + 2}{x^2 - 1}$

58. $f(x) = \frac{x^4}{x^2 + x - 6}$

59. $f(t) = \sqrt{t + 1}$

60. $g(t) = \sqrt{t^2 + 9}$

61. $f(t) = \sqrt[3]{t - 1}$

62. $g(x) = \sqrt{7 - 3x}$

63. $f(x) = \sqrt{1 - 2x}$

64. $g(x) = \sqrt{x^2 - 4}$

65. $g(x) = \frac{\sqrt{2 + x}}{3 - x}$

66. $g(x) = \frac{\sqrt{x}}{2x^2 + x - 1}$

67. $g(x) = \sqrt[4]{x^2 - 6x}$

68. $g(x) = \sqrt{x^2 - 2x - 8}$

69. $f(x) = \frac{3}{\sqrt{x - 4}}$

70. $f(x) = \frac{x^2}{\sqrt{6 - x}}$

71. $f(x) = \frac{(x + 1)^2}{\sqrt{2x - 1}}$

72. $f(x) = \frac{x}{\sqrt[4]{9 - x^2}}$

73–76 ■ Four Ways to Represent a Function A verbal description of a function is given. Find (a) algebraic, (b) numerical, and (c) graphical representations for the function.

73. To evaluate $f(x)$, divide the input by 3 and add $\frac{2}{3}$ to the result.

74. To evaluate $g(x)$, subtract 4 from the input and multiply the result by $\frac{3}{4}$.

75. Let $T(x)$ be the amount of sales tax charged in Lemon County on a purchase of x dollars. To find the tax, take 8% of the purchase price.

76. Let $V(d)$ be the volume of a sphere of diameter d . To find the volume, take the cube of the diameter, then multiply by π and divide by 6.

SKILLS Plus

77–78 ■ Domain and Range Find the domain and range of f .

77. $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 5 & \text{if } x \text{ is irrational} \end{cases}$

78. $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 5x & \text{if } x \text{ is irrational} \end{cases}$

APPLICATIONS

79. **Torricelli's Law** A tank holds 50 gal of water, which drains from a leak at the bottom, causing the tank to empty in 20 min. The tank drains faster when it is nearly full because the pressure on the leak is greater. **Torricelli's Law** gives the volume of water remaining in the tank after t minutes as

$$V(t) = 50 \left(1 - \frac{t}{20} \right)^2 \quad 0 \leq t \leq 20$$

- Find $V(0)$ and $V(20)$.
- What do your answers to part (a) represent?
- Make a table of values of $V(t)$ for $t = 0, 5, 10, 15, 20$.
- Find the net change in the volume V as t changes from 0 min to 20 min.



80. Area of a Sphere The surface area S of a sphere is a function of its radius r given by

$$S(r) = 4\pi r^2$$

- Find $S(2)$ and $S(3)$.
- What do your answers in part (a) represent?

- 81. Relativity** According to the Theory of Relativity, the length L of an object is a function of its velocity v with respect to an observer. For an object whose length at rest is 10 m, the function is given by

$$L(v) = 10\sqrt{1 - \frac{v^2}{c^2}}$$

where c is the speed of light (300,000 km/s).

- (a) Find $L(0.5c)$, $L(0.75c)$, and $L(0.9c)$.
 (b) How does the length of an object change as its velocity increases?
- 82. Pupil Size** When the brightness x of a light source is increased, the eye reacts by decreasing the radius R of the pupil. The dependence of R on x is given by the function

$$R(x) = \sqrt{\frac{13 + 7x^{0.4}}{1 + 4x^{0.4}}}$$

where R is measured in millimeters and x is measured in appropriate units of brightness.

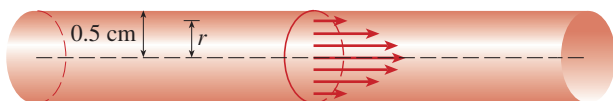
- (a) Find $R(1)$, $R(10)$, and $R(100)$.
 (b) Make a table of values of $R(x)$.
 (c) Find the net change in the radius R as x changes from 10 to 100.



- 83. Blood Flow** As blood moves through a vein or an artery, its velocity v is greatest along the central axis and decreases as the distance r from the central axis increases (see the figure). The formula that gives v as a function of r is called the **law of laminar flow**. For an artery with radius 0.5 cm, the relationship between v (in cm/s) and r (in cm) is given by the function

$$v(r) = 18,500(0.25 - r^2) \quad 0 \leq r \leq 0.5$$

- (a) Find $v(0.1)$ and $v(0.4)$.
 (b) What do your answers to part (a) tell you about the flow of blood in this artery?
 (c) Make a table of values of $v(r)$ for $r = 0, 0.1, 0.2, 0.3, 0.4, 0.5$.
 (d) Find the net change in the velocity v as r changes from 0.1 cm to 0.5 cm.



- 84. How Far Can You See?** Because of the curvature of the earth, the maximum distance D that you can see from the top of a tall building or from an airplane at height h is given by the function

$$D(h) = \sqrt{2rh + h^2}$$

where $r = 3960$ mi is the radius of the earth and D and h are measured in miles.

- (a) Find $D(0.1)$ and $D(0.2)$.
 (b) How far can you see from the observation deck of Toronto's CN Tower, 1135 ft above the ground?
 (c) Commercial aircraft fly at an altitude of about 7 mi. How far can the pilot see?
 (d) Find the net change in the value of distance D as h changes from 1135 ft to 7 mi.
- 85. Income Tax** In a certain country, income tax T is assessed according to the following function of income x :

$$T(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq 10,000 \\ 0.08x & \text{if } 10,000 < x \leq 20,000 \\ 1600 + 0.15x & \text{if } 20,000 < x \end{cases}$$

- (a) Find $T(5,000)$, $T(12,000)$, and $T(25,000)$.
 (b) What do your answers in part (a) represent?
- 86. Internet Purchases** An Internet bookstore charges \$15 shipping for orders under \$100 but provides free shipping for orders of \$100 or more. The cost C of an order is a function of the total price x of the books purchased, given by

$$C(x) = \begin{cases} x + 15 & \text{if } x < 100 \\ x & \text{if } x \geq 100 \end{cases}$$

- (a) Find $C(75)$, $C(90)$, $C(100)$, and $C(105)$.
 (b) What do your answers in part (a) represent?
- 87. Cost of a Hotel Stay** A hotel chain charges \$75 each night for the first two nights and \$50 for each additional night's stay. The total cost T is a function of the number of nights x that a guest stays.

- (a) Complete the expressions in the following piecewise defined function.

$$T(x) = \begin{cases} \text{ } & \text{if } 0 \leq x \leq 2 \\ \text{ } & \text{if } x > 2 \end{cases}$$

- (b) Find $T(2)$, $T(3)$, and $T(5)$.
 (c) What do your answers in part (b) represent?
- 88. Speeding Tickets** In a certain state the maximum speed permitted on freeways is 65 mi/h, and the minimum is 40 mi/h. The fine F for violating these limits is \$15 for every mile above the maximum or below the minimum.
- (a) Complete the expressions in the following piecewise defined function, where x is the speed at which you are driving.

$$F(x) = \begin{cases} \text{ } & \text{if } 0 < x < 40 \\ \text{ } & \text{if } 40 \leq x \leq 65 \\ \text{ } & \text{if } x > 65 \end{cases}$$

- (b) Find $F(30)$, $F(50)$, and $F(75)$.
 (c) What do your answers in part (b) represent?