MATH 195

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§1.4 RATIONAL EXPRESSIONS

11, 12, 13, 17, 19, 27, 33, 35, 43, 45, 49, 50, 57, 62, 69, 73, 81

11. $\sqrt{x+3}$: RADICAND CANNOT BE NEGATIVE

$x + 3 \geq 0$

$x \geq -3$ i.e. $[-3, \infty)$

12. $\frac{1}{\sqrt{x-1}}$ TWO THINGS: (1) RADICAND CANNOT BE NEGATIVE $x - 1 \geq 0 \Rightarrow x \geq 1$

(2) $\sqrt{x-1} \neq 0 \Rightarrow x \neq 1$ DENOMINATOR CANNOT EQUAL ZERO

$\therefore$ $x > 1$ i.e. $(1, \infty)$

13. $\frac{x^2 + 1}{x^2 - x - 2}$ POLYNOMIALS ARE DEFINED FOR ALL $x \in \mathbb{R}$

BUT DENOMINATOR CANNOT EQUAL ZERO:

$x^2 - x - 2 \neq 0$

$(x - 2)(x + 1) \neq 0$

$x - 2 \neq 0$ $\quad$ $x + 1 \neq 0$

$x \neq 2$, $\quad x \neq -1$

i.e. $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

17. $$\frac{x-2}{x^2-4} = \frac{\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = \boxed{\frac{1}{x+2}}$$

19. $$\frac{x^2+5x+6}{x^2+8x+15} = \frac{\cancel{(x+3)}(x+2)}{\cancel{(x+3)}(x+5)} = \boxed{\frac{x+2}{x+5}}$$

27. $$\frac{x^2+2x-15}{x^2-25} \cdot \frac{x-5}{x+2} = \frac{\cancel{(x+5)}(x-3)}{\cancel{(x+5)}\cancel{(x-5)}} \cdot \frac{\cancel{(x-5)}}{(x+2)}$$

$$= \boxed{\frac{x-3}{x+2}}$$

33. $$\frac{x+3}{4x^2-9} \div \frac{x^2+7x+12}{2x^2+7x-15} = \frac{x+3}{4x^2-9} \cdot \frac{2x^2+7x-15}{x^2+7x+12}$$

$$= \frac{\cancel{(x+3)}}{(2x+3)\cancel{(2x-3)}} \cdot \frac{\cancel{(2x-3)}(x+5)}{(x+4)\cancel{(x+3)}} = \frac{(x+5)}{(2x+3)(x+4)}$$

$$= \boxed{\frac{x+5}{2x^2+11x+12}}$$

35. $$\frac{x^3}{x+1} \div \frac{x}{x^2+2x+1} = \frac{x^3}{x+1} \cdot \frac{x^2+2x+1}{x}$$

$$= \frac{x^3(x^2+2x+1)}{(x+1)x} = \frac{x^5+2x^4+x^3}{x^2+x}$$

43. $$\frac{3}{x+1} - \frac{1}{x+2} \qquad LCD = (x+1)(x+2)$$

$$= \frac{3}{x+1} \cdot \frac{x+2}{x+2} - \frac{1}{x+2} \cdot \frac{x+1}{x+1}$$

$$= \frac{3(x+2)-(x+1)}{(x+1)(x+2)} = \boxed{\frac{2x+5}{x^2+3x+2}}$$

45. $$\frac{5}{2x-3} - \frac{3}{(2x-3)^2} \qquad LCD = (2x-3)^2$$

$$= \frac{5(2x-3)}{(2x-3)^2} - \frac{3}{(2x-3)^2} = \boxed{\frac{10x-18}{4x^2-12x+9}}$$

49. $$\frac{1}{x^2} + \frac{1}{x^2+x} = \frac{1}{x^2} + \frac{1}{x(x+1)} \qquad LCD = x^2(x+1)$$

$$\frac{1}{x^2} \cdot \frac{x+1}{x+1} + \frac{1}{x(x+1)} \cdot \frac{1}{x} = \frac{x+1+1}{x^2(x+1)} = \boxed{\frac{x+2}{x^3+x^2}}$$

50. $\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$ $\quad LCD = x^3$

$$\frac{x^2}{x(x^2)} + \frac{x}{x^2(x)} + \frac{1}{x^3} = \boxed{\frac{x^2 + x + 1}{x^3}}$$

57. $$\frac{1}{x^2 + 3x + 2} - \frac{1}{x^2 - 2x - 3} = \frac{1}{(x+2)(x+1)} - \frac{1}{(x-3)(x+1)}$$

$LCD = (x+1)(x+2)(x-3)$ $\quad \frac{(x-3)}{(x+2)(x+1)(x-3)} - \frac{(x+2)}{(x-3)(x+1)(x+2)}$

$$= \boxed{\frac{-5}{(x+2)(x+1)(x-3)}}$$

62. $$\left(1 + \frac{1}{c-1}\right) \div \left(1 - \frac{1}{c-1}\right) = \left(\frac{c-1}{c-1} + \frac{1}{c-1}\right) \div \left(\frac{c-1}{c-1} - \frac{1}{c-1}\right)$$

$$= \frac{c}{c-1} \div \frac{c-2}{c-1} = \frac{c}{\cancel{c-1}} \cdot \frac{\cancel{c-1}}{c-2} = \boxed{\frac{c}{c-2}}$$

69. $$\frac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}} \cdot \frac{x^2 y^2}{x^2 y^2} = \boxed{\frac{y^2 - x^2}{xy^2 + x^2 y}}$$

73. $$\left(\frac{1}{1+x+h} - \frac{1}{1+x}\right)\frac{1}{h} = \frac{(1+x)-(1+x+h)}{(1+x+h)(1+x)} \cdot \frac{1}{h}$$

$$= \frac{-h}{(1+x+h)(1+x)} \cdot \frac{1}{h} = \boxed{\frac{-1}{(1+x+h)(1+x)}}$$

81. $$\frac{2(1+x)^{1/2} - x(1+x)^{-1/2}}{1+x} = \frac{(1+x)^{-1/2}\left[2(1+x) - x\right]}{(1+x)}$$

$$= \boxed{\frac{2+x}{(1+x)^{3/2}}}$$