Name: *Answer Key* (40 points total) 4/10/2019

Math 19500-FG Precalculus **Quiz 3**

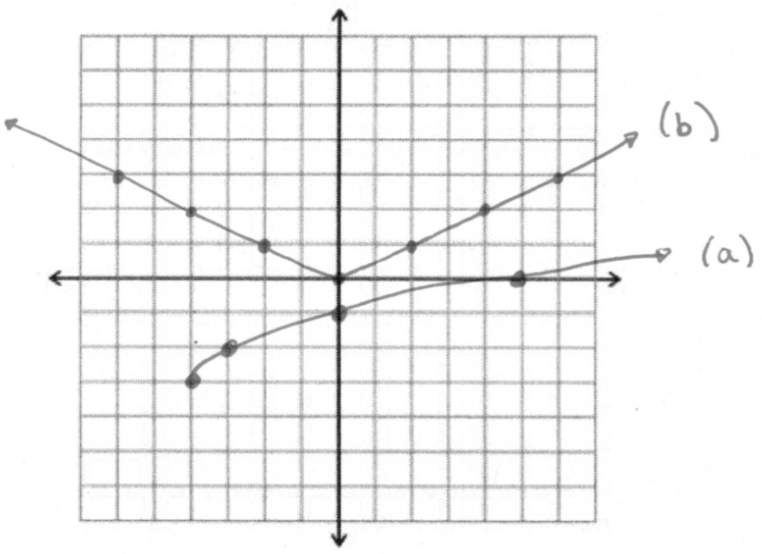
Please show all work and **box your final answers**. If you need more room, you may use the backs of the pages. Calculators are not allowed and cellphones should be put away. Good luck!

1. (4 points) Find the average rate of change of the function $f(x) = \dfrac{x+1}{x-1}$ over the interval $[2,4]$.

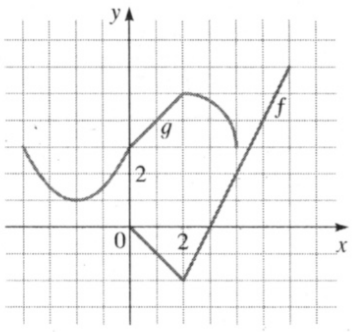
$$\frac{f(4)-f(2)}{4-2} = \frac{\frac{4+1}{4-1} - \frac{2+1}{2-1}}{4-2} = \frac{\frac{5}{3} - 3}{2}$$

$$= \frac{\frac{-4}{3}}{2} = \frac{-4}{3} \cdot \frac{1}{2} = \boxed{-\frac{2}{3}}$$

2. On the axes below, sketch the following graphs.
   (a) (4 points) $y = \sqrt{x+4} - 3$: $y = \sqrt{x}$ SHIFTED LEFT 4, DOWN 3
   (b) (4 points) $y = \frac{1}{2}|x|$: $y = |x|$ WITH y-COORD SCALED BY $\frac{1}{2}$



3. Use the given graphs of f and g to evaluate the expressions.



(a) (2 points) $f \circ g(2)$ $g(2) = 5$, $f(g(2)) = f(5) = \boxed{4}$

(b) (2 points) $f \circ f(4)$ $f(4) = 2$, $f(f(4)) = f(2) = \boxed{-2}$

(c) (2 points) $g \circ g(-2)$ $g(-2) = 1$, $g(g(2)) = g(1) = \boxed{4}$

4. (6 points) Let $f(x) = \dfrac{3}{x-2}$ and $g(x) = \dfrac{2x+3}{x}$. Find $f \circ g(x)$ and write it in simplest form.

$$f(g(x)) = f\left(\frac{2x+3}{x}\right) = \frac{3}{\frac{2x+3}{x} - 2}$$

$$= \frac{3}{\frac{2x+3}{x} - 2} \cdot \frac{x}{x}$$

$$= \frac{3x}{2x+3-2x} = \frac{3x}{3} = \boxed{x}$$

ALT: NOTE THAT $g(x) = 2 + \frac{3}{x}$

SO $f(g(x)) = \dfrac{3}{2 + \frac{3}{x} - 2} = 3 \cdot \dfrac{x}{3} = \boxed{x}$

5. (6 points) Let f be the one-to-one function $f(x) = \dfrac{x-1}{2x+4}$. Find $f^{-1}(x)$.

Let $y = \dfrac{x-1}{2x+4}$

$y(2x+4) = x-1$

$2xy + 4y = x - 1$

$2xy - x = -4y - 1$

$x(2y-1) = -4y - 1$

$x = \dfrac{-4y-1}{2y-1}$

$\therefore \boxed{f^{-1}(x) = \dfrac{-4x-1}{2x-1} \text{ OR } \dfrac{4x+1}{1-2x}}$

6. (4 points) Find the coordinates of the vertex of the parabola with equation $y = 4x - x^2 - 3$. Is this point a local maximum or a local minimum?

GIVEN $y = \underbrace{ax^2+bx+c}_{f(x)} \longrightarrow$ VERTEX $= \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

WE HAVE $y = -x^2 + 4x - 3$

$a = -1 \quad b = 4 \quad c = -3$

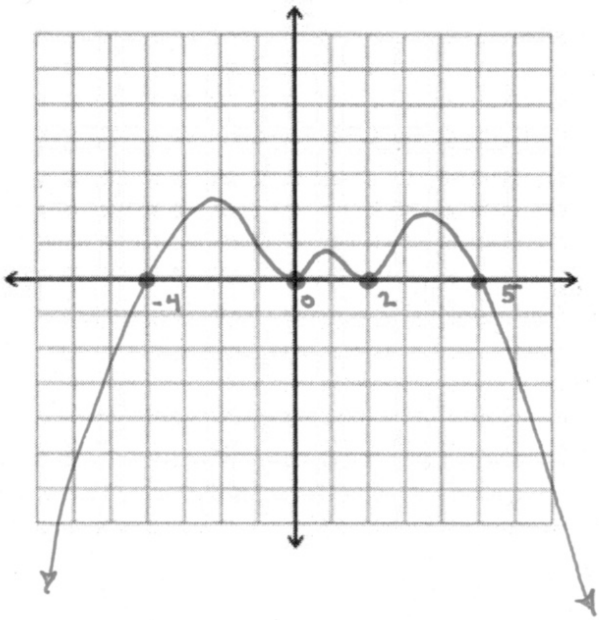
VERTEX: $\left(\frac{-(4)}{2(-1)}, f\left(\frac{-(4)}{2(-1)}\right)\right) = (2, f(2))$

$= (2, -(2)^2 + 4(2) - 3) = (2, -4 + 8 - 3)$

$= \boxed{(2, 1), \text{ MAX}}$

BECAUSE $a < 0$

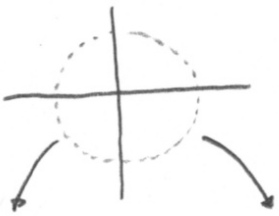
7. (6 points) On the axes below, sketch the graph of the polynomial $f(x) = -x^2(x+4)(x-2)^2(x-5)$. Your sketch should accurately show the intercepts and the intervals on which f is positive/negative.



f IS DEGREE $2+1+2+1 = 6$, EVEN

WITH LEAD COEFFICIENT $(-1)(1)(1)(1) = -1$, NEG.

∴ END BEHAVIOR



FACTOR	POWER	X-INT.	CROSS/BOUNCE
$-x^2$	EVEN	0	BOUNCE
$x+4$	ODD	-4	CROSS
$(x-2)^2$	EVEN	2	BOUNCE
$x-5$	ODD	5	CROSS