Name: *ANSWER KEY* 3/27/2019

**Math 19500-FG Precalculus** **Midterm Exam**

Please show all work and **box your final answers**. If you need more room, you may use the backs of the pages. Calculators are not allowed and cellphones should be put away. Good luck!

1. Evaluate the following expressions. Express your answer in simplest form.

   (a) (4 points) $4\left(\frac{1}{4}+\frac{1}{3}\right)^2$

   $= 4\left(\frac{1}{4}\cdot\frac{3}{3}+\frac{1}{3}\cdot\frac{4}{4}\right)^2 = 4\left(\frac{3}{12}+\frac{4}{12}\right)^2 = 4\left(\frac{7}{12}\right)^2$

   $= 4\left(\frac{49}{144}\right) = \frac{\cancel{4}}{1}\cdot\frac{49}{\cancel{4}\cdot 36} = \boxed{\frac{49}{36}}$

   (b) (4 points) $16^{-3/2}$

   $= \left(\left(16^{1/2}\right)^3\right)^{-1} = \left(4^3\right)^{-1} = 64^{-1} = \boxed{\frac{1}{64}}$

2. (4 points) Use interval notation to describe the domain of the function $f(x) = \dfrac{x^2-1}{x\sqrt{x+2}}$.

i) CANNOT TAKE $\sqrt{\ }$ OF NEG. #'s : $x+2 \geq 0$

$x \geq -2$

ii) CANNOT DIVIDE BY 0 : $x\sqrt{x+2} \neq 0$

$x \neq 0$ $x \neq -2$


$\boxed{(-2,0) \cup (0,\infty)}$

3. Factor the following expressions completely.

(a) (4 points) $4x^5 + 24x^4 - 64x^3$

$= 4x^3(x^2 + 6x - 16)$

$= \boxed{4x^3(x+8)(x-2)}$

(b) (4 points) $\underbrace{x^3 - 4x^2} - \underbrace{4x + 16}$

$= x^2(x-4) - 4(x-4)$

$= (x^2-4)(x-4)$

$= \boxed{(x+2)(x-2)(x-4)}$

(c) (4 points) $x^4 - 10x^2 + 9$   LET $w = x^2$, $w^2 = x^4$

$= w^2 - 10w + 9$

$= (w-9)(w-1)$

$= (x^2-9)(x^2-1)$

$= \boxed{(x+3)(x-3)(x+1)(x-1)}$

4. Solve the following equations.

(a) (4 points) $|3x + 5| = 1$

$3x + 5 = 1$ OR $3x + 5 = -1$

$3x = -4$ $\qquad$ $3x = -6$

$\boxed{x = -\frac{4}{3}}$ $\qquad$ $\boxed{x = -2}$

(b) (4 points) $\frac{3}{x+1} - \frac{1}{2} = \frac{1}{3x+3}$ $\qquad$ LCD $= 2 \cdot 3 (x+1) = 6(x+1)$

$3(x+1)$

$\frac{3}{x+1} \cdot 6(x+1) - \frac{1}{2} \cdot 6(x+1) = \frac{1}{3(x+1)} \cdot 6(x+1)$

$18 - 3(x+1) = 2$ $\qquad$ $13 = 3x$

$18 - 3x - 3 = 2$ $\qquad$ $\boxed{x = \frac{13}{3}}$

5. Let $\overline{AB}$ be the line segment connecting the points $A(2, -3)$ and $B(1, 4)$.

(a) (4 points) Find the midpoint of $\overline{AB}$. $\qquad$ $x_1 \; y_1 \quad x_2 \; y_2$

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{2+1}{2}, \frac{-3+4}{2} \right)$$

$$= \boxed{\left( \frac{3}{2}, \frac{1}{2} \right)}$$

(b) (4 points) Find the length of $\overline{AB}$.

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(2-1)^2 + (-3-4)^2}$$

$$= \sqrt{1 + 49} = \sqrt{50} = \sqrt{25}\sqrt{2} = \boxed{5\sqrt{2}}$$

6. Solve the following inequalities.

(a) (4 points) $4 \leq 3x - 2 < 13$

$+2 \quad +2 \quad +2$

$\frac{6}{3} \leq \frac{3x}{3} < \frac{15}{3}$

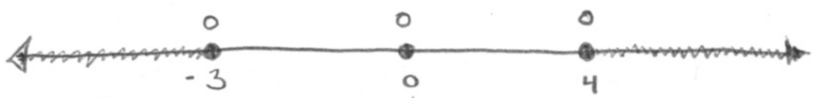
$2 \leq x < 5$

$\boxed{[2, 5)}$

(b) (6 points) $x^2(x+3)(x-4) \geq 0$

ZEROS: $0, -3, 4$

$x^2(x+3)(x-4)$




| | | | | |
|---|---|---|---|---|
| $x^2$ | + | + | + | + |
| $x+3$ | − | + | + | + |
| $x-4$ | − | − | − | + |
| $x^2(x+3)(x-4)$ | + | − | − | + |

$\boxed{(-\infty, -3] \cup \{0\} \cup [4, \infty)}$

(c) (6 points) $\frac{1}{x+1} + \frac{1}{x+2} \leq 0$

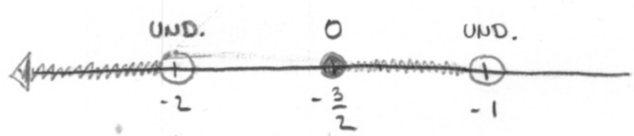
NOTE: $x \neq -1$ , $x \neq -2$

$\frac{1}{x+1} \cdot \frac{x+2}{x+2} + \frac{1}{x+2} \cdot \frac{x+1}{x+1} \leq 0 \longrightarrow \frac{2x+3}{(x+1)(x+2)} \leq 0$

ZEROS: $2x+3 = 0 \longrightarrow x = -3/2$

$x+1 = 0 \longrightarrow x = -1$

$x+2 = 0 \longrightarrow x = -2$




| | | | | |
|---|---|---|---|---|
| $2x+3$ | − | − | + | + |
| $x+1$ | − | − | − | + |
| $x+2$ | − | + | + | + |
| $\frac{2x+3}{(x+1)(x+2)}$ | − | + | − | + |

$\boxed{(-\infty, -2) \cup [-\frac{3}{2}, -1)}$

7. Let

$$f(x) = \frac{1}{x} \qquad \text{and} \qquad g(x) = x^2 - 1.$$

(a) (6 points) Evaluate and simplify $\dfrac{g(a+h) - g(a)}{h}$

$g(a+h) = (a+h)^2 - 1$

$= a^2 + 2ah + h^2 - 1$

$g(a) = a^2 - 1$

$$= \frac{[\cancel{a^2} + 2ah + h^2 \cancel{-1}] - [\cancel{a^2} \cancel{-1}]}{h}$$

$$= \frac{\cancel{h}(2a+h)}{\cancel{h}} = \boxed{2a+h}$$

(b) (4 points) Evaluate $f \circ g(x) = f(g(x))$.

$$f(g(x)) = \begin{cases} f(x^2-1) \\ \dfrac{1}{g(x)} \end{cases} = \boxed{\frac{1}{x^2-1}}$$

(BOTH)

(c) (4 points) Evaluate $f \circ g \circ f(x) = f(g(f(x)))$.

SINCE $f \circ g(x) = \dfrac{1}{x^2 - 1}$ (PART (b))

$$f \circ g \circ f(x) = f \circ g(f(x)) = \frac{1}{f(x)^2 - 1} = \boxed{\frac{1}{\left(\frac{1}{x}\right)^2 - 1} \quad \text{OR} \quad \frac{x^2}{1 - x^2}}$$

(d) (4 points) What is the average rate of chance of $f$ over $[1, 5]$?

$$\frac{f(5) - f(1)}{5 - 1} = \frac{\frac{1}{5} - 1}{4} = \frac{-\frac{4}{5}}{4} = \boxed{-\frac{1}{5}}$$

$\left( \text{i.e. } -\frac{4}{5} \cdot \frac{1}{4} \right)$

8. (6 points) Find the center and radius of the circle with equation $x^2 + y^2 - 4x - 2y = 4$.

$$x^2 - 4x \quad + y^2 - 2y \quad = 4$$

$$\left( x^2 + bx + \underline{\left(\tfrac{b}{2}\right)^2} = \left(x + \tfrac{b}{2}\right)^2 \right)$$

IDENTITY

$$x^2 - 4x + \underline{4} + y^2 - 2y + \underline{1} = 4 + \underline{4} + \underline{1}$$

$$(x-2)^2 + (y-1)^2 = 9 = 3^2$$

$$\boxed{\begin{array}{ll} \text{CENTER}: & (2,1) \\ \text{RADIUS}: & 3 \end{array}}$$

9. (a) (4 points) Find an equation for the line with $x$-intercept $-3$ and $y$-intercept 4.

$(-3, 0)$    $x_1 \; y_1$

$(0, 4)$    $x_2 \; y_2$

SLOPE $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 0}{0 - (-3)} = \dfrac{4}{3}$

$\boxed{y = \frac{4}{3}x + 4}$ (SLOPE - INTERCEPT EQ)

(b) (4 points) Find an equation for the line the passes through the point $(5, -1)$ and is parallel to the line with equation $6x - 3y = 2$.

$x_1 \; y_1$

$$-3y = -6x + 2$$

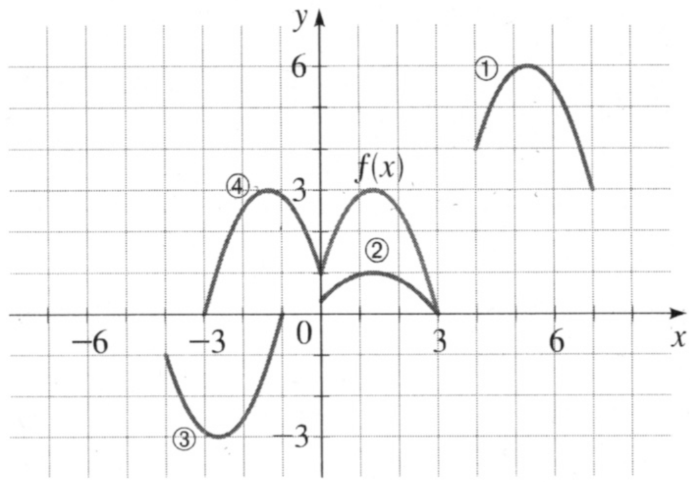
$$y = 2x - \frac{2}{3} \Rightarrow \text{SLOPE } m = 2$$

POINT - SLOPE EQ: $\boxed{\begin{array}{l} y - (-1) = 2(x - 5) \\ \text{OR} \quad y = 2x - 11 \end{array}}$

10. (4 points) The graph $y = f(x)$ is shown below, along with 4 other graphs labeled 1-4. Match each of the following equations with its graph (1, 2, 3, or 4).

**(a)** $y = \frac{1}{3}f(x)$ 2 **(b)** $y = -f(x+4)$ 3

**(c)** $y = f(x-4)+3$ 1 **(d)** $y = f(-x)$ 4




11. (8 points) Sketch the graph $y = -2\sqrt{-x} + 4$