Math 19000-1XB Exam 1 Summer 2022

Name: *ANSWER KEY* 6/23/2022

Answer each question in the space provided and write your final answer to each question on the answer line. If you need more room, you can continue your work on the back of the page. Good luck!

1. (5 points) Express the inequality $x \geq -3$ in interval notation.

1. $[-3, \infty)$

2. (5 points) Evaluate the expression numerically.

$$\frac{\frac{2}{5}+\frac{1}{2}}{\frac{1}{10}+\frac{3}{15}}$$

$= \left(\frac{2}{5}+\frac{1}{2}\right) \div \left(\frac{1}{10}+\frac{3}{15}\right) = \left[\frac{2}{5}\left(\frac{2}{2}\right)+\frac{1}{2}\left(\frac{5}{5}\right)\right] \div \left[\frac{1}{10}\left(\frac{3}{3}\right)+\frac{3}{15}\left(\frac{2}{2}\right)\right]$

GET COMMON DENOMINATORS

$= \left(\frac{4}{10}+\frac{5}{10}\right) \div \left(\frac{3}{30}+\frac{6}{30}\right)$

$= \frac{9}{10} \div \frac{9}{30} = \frac{9}{10} \times \frac{30}{9} = \frac{30}{10} = 3$

OR: MULTIPLY TOP & BOTTOM BY LCD

$\frac{\frac{2}{5}+\frac{1}{2}}{\frac{1}{10}+\frac{3}{15}} \times \frac{30}{30}$

$= \frac{12+15}{3+6} = \frac{27}{9} = 3$

2. 3

3. (5 points) Simplify the expression and eliminate any negative exponents.

$$\left(\frac{2x^3y^{-4}}{3y^{-1}z^{-5}}\right)^{-2}$$

$= \left(\frac{2x^3z^5}{3y^3}\right)^{-2} = \left(\frac{3y^3}{2x^3z^5}\right)^{2} = \frac{3^2(y^3)^2}{2^2(x^3)^2(z^5)^2}$

$= \frac{9y^6}{4x^6z^{10}}$

3.

4. (5 points) Evaluate the expression numerically.

$$\left(\sqrt[4]{6}\right)^{-8} + \frac{\sqrt{75}}{\sqrt{3}}$$

$= (6^{1/4})^{-8} + \frac{\sqrt{25}\sqrt{3}}{\sqrt{3}} = 6^{-2} + 5$

$= \frac{1}{36} + 5 = \frac{1}{36} + \frac{180}{36}$ COMMON DENOM.

4. $\frac{181}{36}$

5. (5 points) Simplify the expression.

$$x^{5/2}(\sqrt{x} - \frac{1}{\sqrt{x}})$$

$= x^{5/2}(x^{1/2} - x^{-1/2}) = x^{5/2+1/2} - x^{5/2-1/2}$

$= x^{6/2} - x^{4/2}$

5. $x^3 - x^2$

6. (5 points) Simplify the expression.

$$\left(w - \frac{1}{w}\right)^{-2}$$

$= \left(\frac{w^2-1}{w}\right)^{-2} = \left(\frac{w}{w^2-1}\right)^2$

$= \frac{w^2}{(w^2-1)^2} =$

6. $\frac{w^2}{w^4 - 2w^2 + 1}$

7. (5 points) Perform the indicated operations and simplify.

$$2(x-1)(3x+3) - 3x(2x-1)$$

DISTRIBUT NEG. SIGN TOO!

$= 2(3x^2 - 3) - 6x^2 + 3x$

$= 6x^2 - 6 - 6x^2 + 3x$

7. $3x - 6$

8. (5 points) Perform the indicated operations and simplify.

$$\left(t - \frac{3}{t}\right)^2$$

$(a-b)^2 = a^2 - 2ab + b^2$

$= t^2 - 2(t)\left(\frac{3}{t}\right) + \left(\frac{3}{t}\right)^2$

8. $t^2 - 6 + \frac{9}{t^2}$

9. (5 points) Factor completely.

$$3(t+6)^2 + 6t(t+6)$$

$3(t+6)\left[(t+6) + 2t\right] = 3(t+6)(3t+6)$

COMMON FACTOR

↓ FACTOR THE FACTOR!

$= 3(t+6)3(t+2)$

9. $9(t+6)(t+2)$

10. (5 points) Factor the expression completely.

$$x^4 + 5x^3 - 24x^2$$

$= x^2(x^2 + 5x - 24)$

COMMON FACTOR

10. $x^2(x+8)(x-3)$

11. (5 points) Factor the expression completely.

$$16x^2 - 25$$

$= (4x)^2 - 5^2$ DIFFERENCE OF SQUARES $a^2 - b^2 = (a+b)(a-b)$

$= (4x+5)(4x-5)$

11. $(4x+5)(4x-5)$

12. (5 points) Perform the indicated operation and simplfy.

$$\frac{10x-2}{x+2} - 2$$

$= \frac{10x-2}{x+2} - 2\left(\frac{x+2}{x+2}\right) = \frac{10x-2-2(x+2)}{x+2}$

GET COMMON DENOMINATORS

$= \frac{10x-2-2x-4}{x+2}$

COMBINE SIMILAR TERMS

12. $\frac{8x-6}{x+2}$

13. (5 points) Perform the indicated operation and simplify.

$$\frac{x^2+4x+4}{x^2-2x+1} \cdot \frac{x^2-6x+5}{x^2-3x-10}$$

FIRST FACTOR $\dfrac{(x+2)^2}{(x-1)^2} \cdot \dfrac{(x-5)(x-1)}{(x-5)(x+2)}$

13. $\dfrac{x+2}{x-1}$

14. (5 points) Solve the equation.

$$\frac{18x-5}{9x+3} = 2 - \frac{3}{x}$$

MULTIPLY EVERY TERM BY LCD: $x(9x+3)$

$x(9x+3)\dfrac{18x-5}{9x+3} = x(9x+3)\,2 - x(9x+3)\dfrac{3}{x}$

$18x^2 - 5x = 18x^2 + 6x - 27x - 9$

$16x = -9$

14. $x = -\dfrac{9}{16}$

15. (5 points) Solve the equation.

$$\frac{x^2-1}{x+2} = \frac{x^2+x-4}{x+3}$$

CROSS MULTIPLY

$(x^2-1)(x+3) = (x+2)(x^2+x-4)$

$x^3 + 3x^2 - x - 3 = x^3 + x^2 - 4x + 2x^2 + 2x - 8$

$3x^2 - x - 3 = 3x^2 - 2x - 8$

$-x + 2x = -8 + 3$

15. $x = -5$

16. (5 points) Solve the equation.

$$\frac{4}{5}w + \frac{1}{4}(w-5) = \frac{w+1}{2}$$

MULTIPLY EVERY TERM BY LCD: 20

$$20\cdot\frac{4}{5}w + 20\cdot\frac{1}{4}(w-5) = 20\cdot\frac{w+1}{2}$$

$$16w + 5(w-5) = 10(w+1)$$

$$16w + 5w - 25 = 10w + 10$$

$$21w - 25 = 10w + 10$$

$$11w = 35$$

16. $w = \frac{35}{11}$

17. (5 points) Find the distance between the points $(-3, 3)$ and $(1, -5)$.

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(1-(-3))^2 + (-5-3)^2}$$

$$= \sqrt{16+64} = \sqrt{80} = \sqrt{16}\sqrt{5}$$

17. $4\sqrt{5}$

18. (5 points) Find the midpoint of the line segment connecting $(2, 1)$ and $(9, -3)$.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2+9}{2}, \frac{1-3}{2}\right)$$

18. $\left(\frac{11}{2}, -1\right)$

19. (5 points) Determine which of the given points are on the graph of the equation.

$$\sqrt{y} = (x-5)^2; \qquad (8,3), (0,25), (4,1), (2,81)$$

$(8,3)$: $\sqrt{3} = (8-5)^2$

$\sqrt{3} = 9$ ✗

$(0,25)$: $\sqrt{25} = (0-5)^2$

$5 = 25$ ✗

$(4,1)$: $\sqrt{1} = (4-5)^2$

$1 = 1$ ✓

$(2,81)$: $\sqrt{81} = (2-5)^2$

$9 = 9$ ✓

19. $(4,1)$ & $(2,81)$

20. (5 points) Give an equation of the circle with center $(3,-4)$ that passes through the origin.

$(x-h)^2 + (y-k)^2 = r^2$

$(x-3)^2 + (y+4)^2 = r^2$

NOTE: r = DISTANCE FROM CENTER $(0,0)$ TO $(3,-4)$

$r = \sqrt{(3-0)^2 + (-4-0)^2}$

$r^2 = (3-0)^2 + (-4-0)^2$

$r^2 = 9 + 16 = 25$

20. $(x-3)^2 + (y+4)^2 = 25$

KNOW THE DIFFERENCE:

(1) $\frac{10x-2}{x+2}-2$ $\qquad$ $\left(w-\frac{1}{w}\right)^{-2}$ $\qquad$ $\frac{\frac{2}{5}+\frac{1}{2}}{\frac{1}{10}+\frac{3}{15}}$

TO **ADD** FRACTIONS, WE NEED COMMON DENOMINATORS.
TRY TO USE LEAST COMMON DENOMINATOR.

(2) $\frac{x^2+4x+4}{x^2-2x+1}\cdot\frac{x^2-6x+5}{x^2-3x-10}$

WHEN **MULTIPLYING** FRACTIONS, WE CAN CANCEL
COMMON FACTORS IN NUMERATOR & DENOMINATOR
(OF EITHER FRACTION) AFTER FACTORING.

EQUATIONS ARE DIFFERENT

(3) $\frac{x^2-1}{x+2}=\frac{x^2+x-4}{x+3}$ $\qquad$ $\frac{4}{5}w+\frac{1}{4}(w-5)=\frac{w+1}{2}$ $\qquad$ $\frac{18x-5}{9x+3}=2-\frac{3}{x}$

WHEN FRACTIONS ARE **EQUAL**, WE CAN MULTIPLY
EVERY TERM ON BOTH SIDES OF EQUAL SIGN
BY LEAST COMMON DENOMINATOR.
BY CONSTRUCTION, EVERY DENOMINATOR WILL CANCEL.

IF THE EQUATION IS VERY SIMPLE – 2 FRACTIONS
EQUAL – THIS IS THE SAME AS "**CROSS MULTIPLYING**"

$$\frac{a}{b}=\frac{c}{d} \iff ad=bc$$

DEFINITIONS:

FACTOR = REWRITE A SUM/DIFFERENCE AS A PRODUCT (OF FACTORS)

e.g. $2t^2 + 6t - 8 = (2t+8)(t-1)$

FACTORED

FACTOR COMPLETELY = REWRITE A SUM/DIFFERENCE AS A PRODUCT (OF FACTORS) AND FACTOR THE FACTORS IF POSSIBLE

e.g. $2t^2 + 6t - 8 = (2t+8)(t-1)$

$= 2(t+4)(t-1)$

FACTORED COMPLETELY

Wrong: $\frac{x^2+4x+4}{x^2-2x+1} \cdot \frac{x^2-6x+5}{x^2-3x-10} = \frac{4x+4}{-2x+1} \cdot \frac{-6x+5}{-3x-10}$

$= ?$

Correct: $\frac{x^2+4x+4}{x^2-2x+1} \cdot \frac{x^2-6x+5}{x^2-3x-10}$

$= \frac{(x+2)(x+2)}{(x-1)(x-1)} \cdot \frac{(x-5)(x-1)}{(x-5)(x+2)}$

$= \frac{x+2}{x-1}$

Wrong: $\underbrace{\frac{a+b}{a+c}}_{\text{addition}} \neq \frac{b}{c}$ e.g. $\underbrace{\frac{101}{102}}_{\approx .9902} = \frac{100+1}{100+2} \neq \frac{1}{2}$

Correct: $\underbrace{\frac{ab}{ac}}_{\text{multiplication}} = \frac{b}{c}$ e.g. $\frac{100}{200} = \frac{100 \cdot 1}{100 \cdot 2} = \frac{1}{2}$