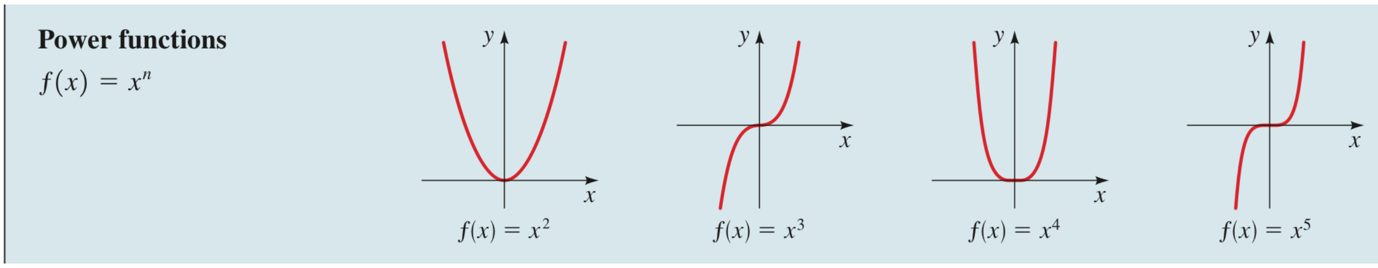
§3.2 Polynomial Functions & Their Graphs

Know the Graphs of Power Functions (aka Monomials)



ex. Sketch the graphs of (a) $y = (x+2)^4$

(b) $y = -\frac{1}{4}x^3$

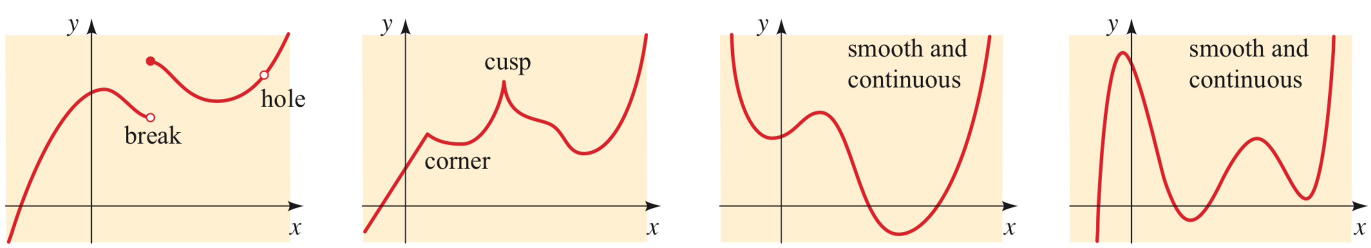
(c) $y = -(x+5)^5 + 1$

Big Idea: Graphs of polynomials $(y = P(x))$ are

- **continuous** - No gaps, holes, jumps, can be drawn without lifting your pencil

and

- **smooth** - No corners or cusps (in calculus you will use the word "differentiable")



Not the graph of a polynomial function

Not the graph of a polynomial function

Graph of a polynomial function

Graph of a polynomial function

BIG IDEA: IF $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ AND $|x|$ IS LARGE,

THEN $f(x) \approx a_n x^n$

LEAD TERM DOMINATES!

e.g.

x	$x^5 + 2x^3 + 2$	x^5
3	299	243
1,000,000	1,000,000,000,000,200,000,000,000,000,002	1,000,000,000,000,000,000,000,000,000,000

DIFFERENCE IS NEGLIGIBLE WHEN X IS LARGE

IMPLICATION: GRAPH OF $y = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$

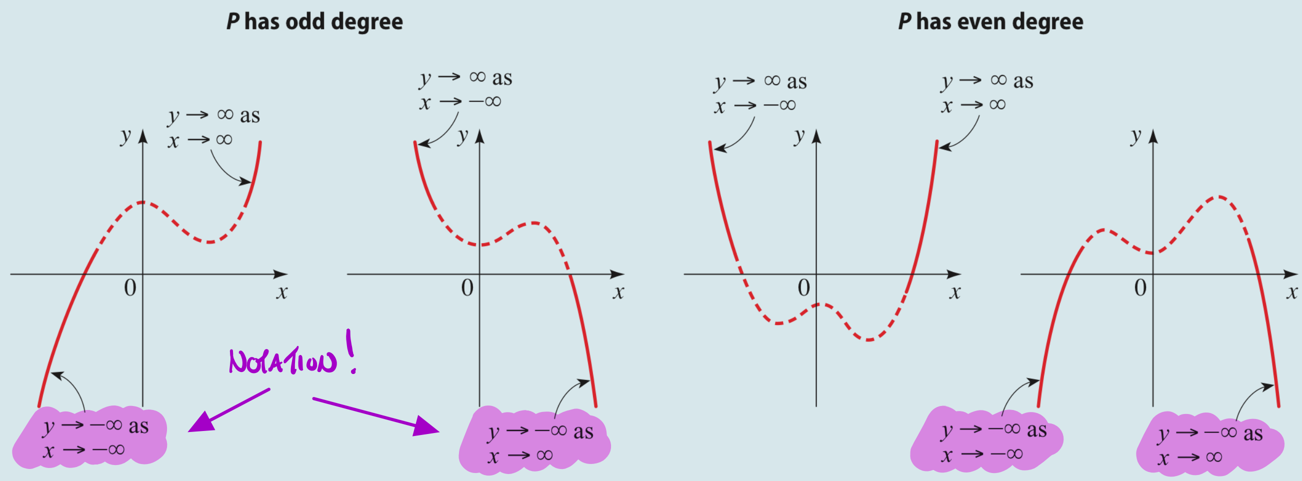
LOOKS LIKE THE GRAPH OF $y = a_n x^n$ WHEN $|x|$ IS LARGE.

i.e. IDENTICAL TAILS

i.e. IDENTICAL END BEHAVIOR.

END BEHAVIOR OF POLYNOMIALS

The end behavior of the polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is determined by the degree n and the sign of the leading coefficient a_n, as indicated in the following graphs.



Leading coefficient positive | Leading coefficient negative | Leading coefficient positive | Leading coefficient negative

ex. DETERMINE THE END BEHAVIOR OF

(a) $y = -\frac{2}{3}x^3 + 5x^2 - \frac{1}{2}x + 7$

(b) $y = -9x^4 + 2x^3 + 7x - 128$

(c) $y = 4(x-1)(x-2)(x-3)(x-4)(x-5)$

REAL ZEROS OF POLYNOMIALS

If P is a polynomial and c is a real number, then the following are equivalent:

1. c is a zero of P.
2. $x = c$ is a solution of the equation $P(x) = 0$.
3. $x - c$ is a factor of $P(x)$.
4. c is an x-intercept of the graph of P.

5. C IS A ROOT OF THE POLYNOMIAL $P(x)$

INTERMEDIATE VALUE THEOREM FOR POLYNOMIALS

If P is a polynomial function and $P(a)$ and $P(b)$ have opposite signs, then there exists at least one value c between a and b for which $P(c) = 0$.

EXAMPLE 5 ■ Finding Zeros and Graphing a Polynomial Function

Let $P(x) = x^3 - 2x^2 - 3x$.

(a) Find the zeros of P. **(b)** Sketch a graph of P.

USE A SIGN CHART TO DETERMINE WHETHER GRAPH IS ABOVE/BELOW X-AXIS IN BETWEEN THE ZEROS (X-INTERCEPTS)

EXAMPLE 6 ■ Finding Zeros and Graphing a Polynomial Function

Let $P(x) = -2x^4 - x^3 + 3x^2$.

(a) Find the zeros of P. **(b)** Sketch a graph of P.

EXAMPLE 7 ■ Finding Zeros and Graphing a Polynomial Function

Let $P(x) = x^3 - 2x^2 - 4x + 8$.

(a) Find the zeros of P. **(b)** Sketch a graph of P.

Def: EACH FACTOR OF A POLYNOMIAL CORRESPONDS TO A ZERO/ROOT OF THE POLYNOMIAL.

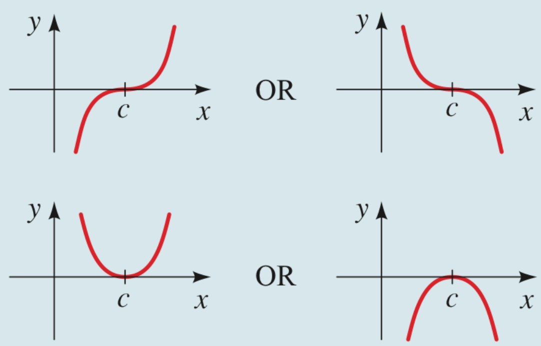
THE EXPONENT ATTACHED TO THAT FACTOR IS CALLED THE MULTIPLICITY OF THE CORRESPONDING ZERO/ROOT.

IT DETERMINES THE SHAPE OF $y = P(x)$ AROUND THE ZERO/ROOT/X-INTERCEPT.

SHAPE OF THE GRAPH NEAR A ZERO OF MULTIPLICITY m

If c is a zero of P of multiplicity m, then the shape of the graph of P near c is as follows.

Multiplicity of c	**Shape of the graph of P near the x-intercept c**
m odd, $m > 1$	
m even, $m > 1$	



ex. GRAPH THE POLYNOMIAL:

26. $P(x) = -(x + 1)^2(x - 1)^3(x - 2)$

27. $P(x) = \frac{1}{12}(x + 2)^2(x - 3)^2$

28. $P(x) = (x - 1)^2(x + 2)^3$

29. $P(x) = x^3(x + 2)(x - 3)^2$

30. $P(x) = (x - 3)^2(x + 1)^2$

40. $P(x) = \frac{1}{8}(2x^4 + 3x^3 - 16x - 24)^2$

41. $P(x) = x^4 - 2x^3 - 8x + 16$

42. $P(x) = x^4 - 2x^3 + 8x - 16$

43. $P(x) = x^4 - 3x^2 - 4$

44. $P(x) = x^6 - 2x^3 + 1$