

EXAM #1 Next Tue. 6/23

4.7. TODAY

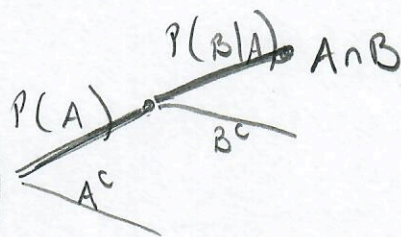
sections

1.5, 2.2-4, 4.2-8, 5.2.

4.40

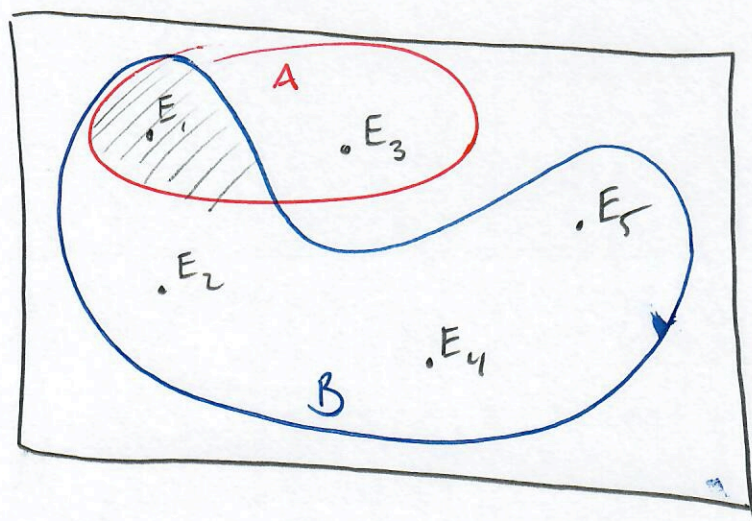
(b).

$$P(A \cap B) = P(A)P(B|A) \\ = P(B)P(A|B)$$



$$\text{IND: } P(A|B) = P(A)$$

$$P(B|A) = P(B)$$



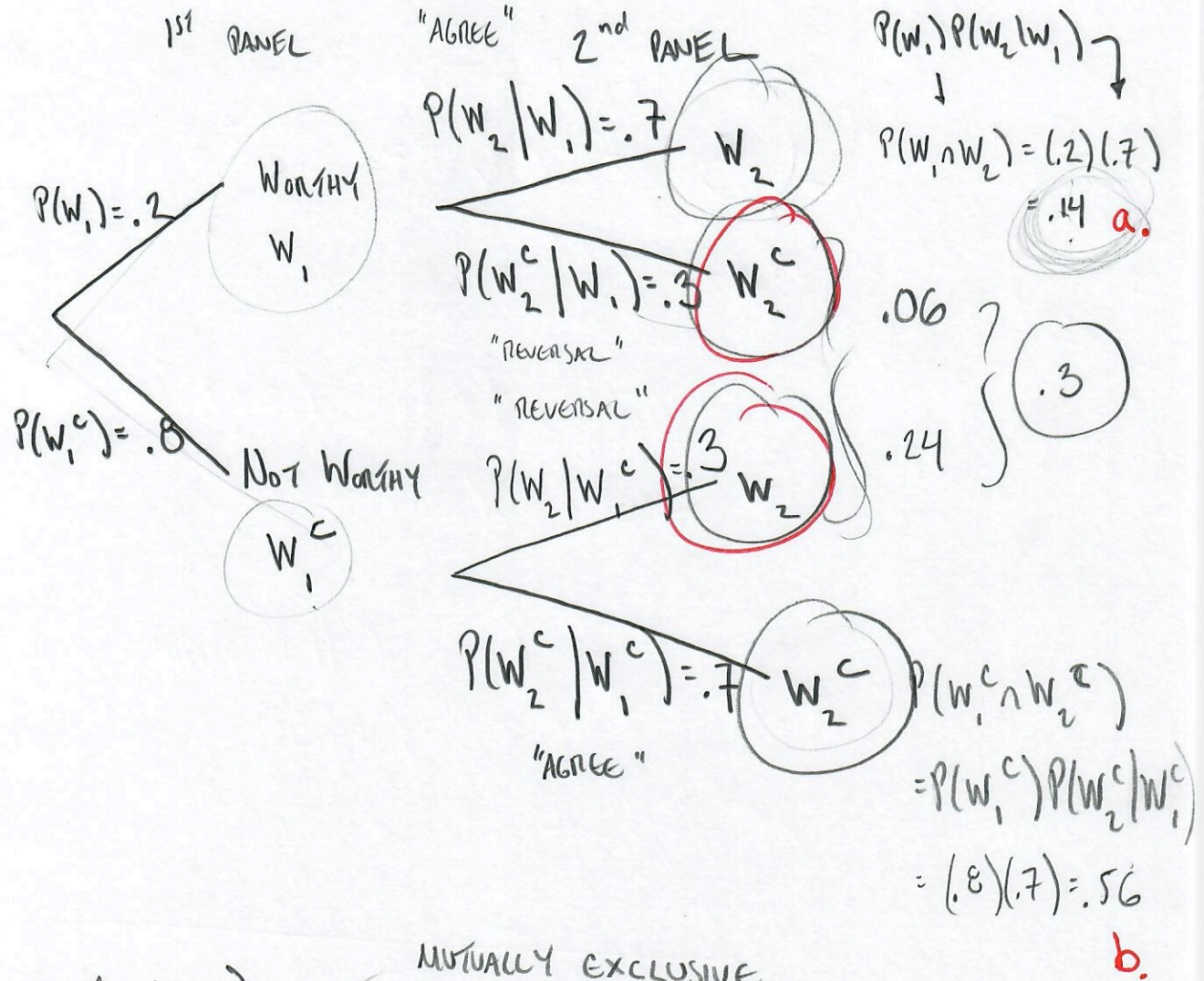
$$A \cap B = \{E_1\}$$

$$P(A \cap B) = P(E_1) = \frac{1}{5} \text{ SAME} = .2$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/5}{4/5} = \frac{1}{4} = .25$$

$$P(A \cap B) = P(B)P(A|B) \\ = (.8)(.25)$$

4.55



MUTUALLY EXCLUSIVE

$$\begin{aligned}
 &P(\text{APPROVED BY 1 PANEL}) \\
 &= P((W_1 \cap W_2^c) \cup (W_1^c \cap W_2)) \\
 &= P(W_1 \cap W_2^c) + P(W_1^c \cap W_2) - P((W_1 \cap W_2^c) \cap (W_1^c \cap W_2)) \\
 &= P(W_1)P(W_2^c|W_1) + P(W_1^c)P(W_2|W_1^c) \\
 &= (.2)(.3) + (.8)(.3) = \boxed{.3}
 \end{aligned}$$

EXAM
PLAN:

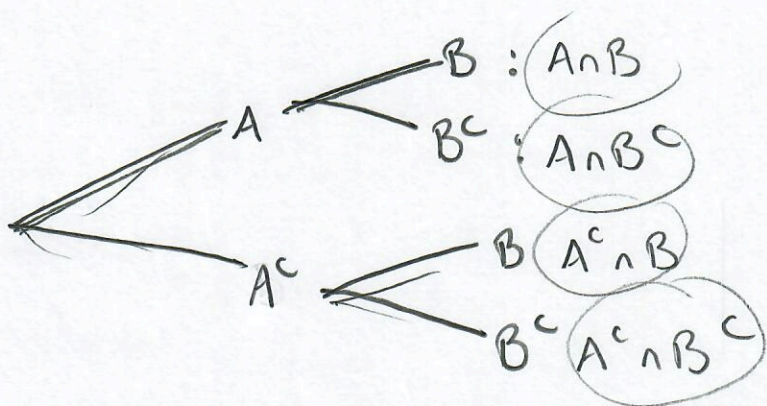
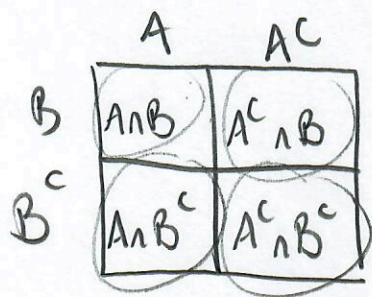
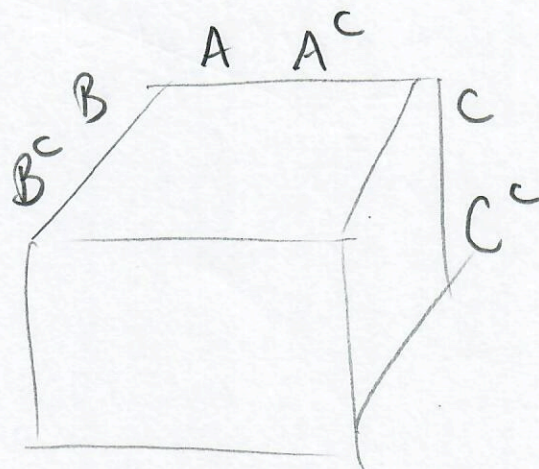
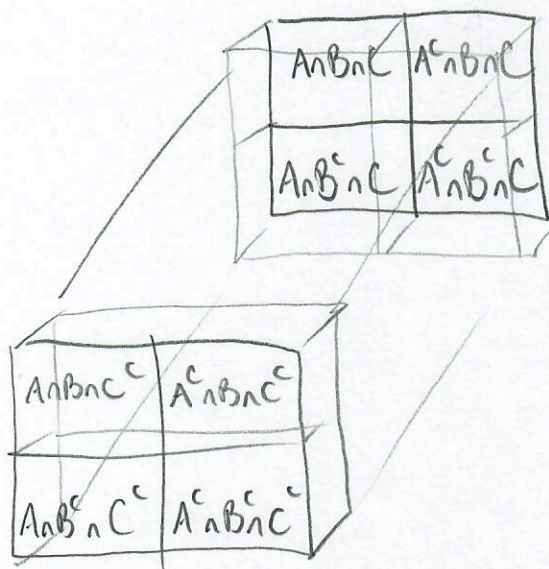
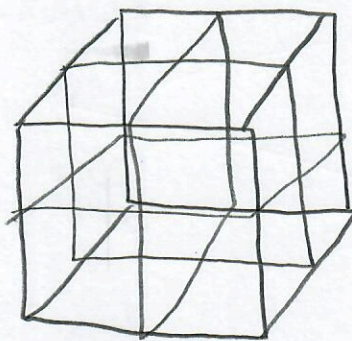
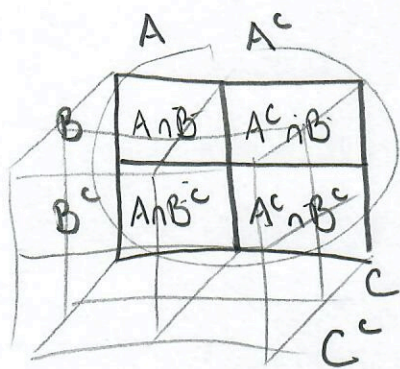
A. WebWork MAA (MAA)

ONLINE MATH PLATFORM (FREE)

B. BLACKBOARD

EMAIL THIS WEEK WITH INSTRUCTIONS

EXAM ONLINE 10:30 - 12:10 PM (TIMED)
Tues. 6/23.



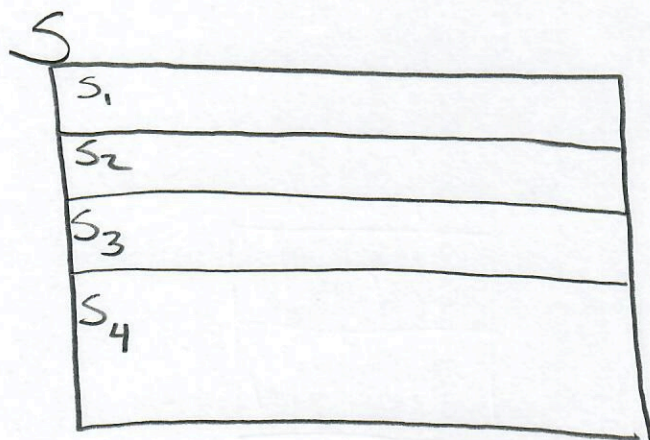
§4.7 BAYES' RULE

1. LAW OF TOTAL PROBABILITY
 2. BAYES' RULE
-

SUPPOSE SAMPLE SPACE CAN BE BROKEN UP INTO k SUBSPACES:
 $S_1, S_2, \dots, S_k$. SUPPOSE THE k SUBSPACES ARE

(1) MUTUALLY EXCLUSIVE

(2) EXHAUSTIVE (TOGETHER, THEIR UNION,
IS THE ENTIRE SAMPLE SPACE)



4 SUBSPACES

$$(1) S_i \cap S_j = \emptyset \quad (i \neq j)$$

$$(2) S_1 \cup S_2 \cup \dots \cup S_k = S$$

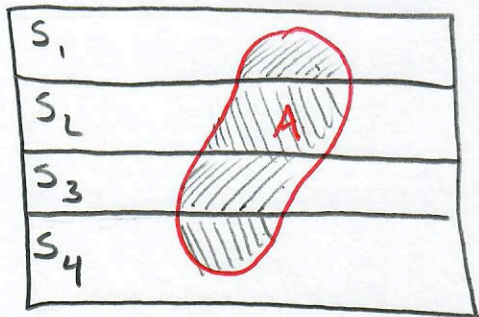
eg. STATE,
EMPLOYMENT STATUS,
...

ALL REGISTERED VOTERS IN U.S.

AGE 18-25	:
AGE 26-40	:
AGE 41-65	:
AGE ≥ 66	:

"DEMOGRAPHICS"

MUTUALLY EXCLUSIVE



$$P(A) = P\left((A \cap S_1) \cup (A \cap S_2) \cup (A \cap S_3) \cup (A \cap S_4)\right)$$

$$= P(A \cap S_1) + P(A \cap S_2) + P(A \cap S_3) + P(A \cap S_4)$$

LAW OF TOTAL PROB: IF $S = S_1 \cup S_2 \cup \dots \cup S_k$,

THE S_i 'S ARE ALL MUTUALLY EXCLUSIVE THEN

FOR ANY EVENT A , $P(A) = P(A \cap S_1) + P(A \cap S_2) + \dots + P(A \cap S_k)$

(RECALL $P(A \cap B) = P(B)P(A|B)$)

$$\therefore P(A) = P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + \dots + P(S_k)P(A|S_k)$$

ex.

EXPERIMENT : SELECT 1 RESIDENT OF NYC AT RANDOM.

SAMPLE SPACE = POPULATION OF NYC

("SUBSPACES" = "SUBPOPULATIONS")

5 SUBSPACES (5 SUBPOPULATIONS)	19% OF NYC RESIDENTS LIVE IN	MANHATTAN
	31% OF	BROOKLYN
	27%	QUEENS
	17%	BRONX
	6%	STATEN ISLAND

(2015)

29% OF MANHATTAN RESIDENTS ARE FOREIGN BORN

38% BK

49% Q

32% BX

21% SI

	S_1 M	S_2 BK	S_3 Q	S_4 BX	S_5 SI
Proportion of NYC	.19	.31	.27	.17	.06
Proportion Foreign Born	.29	.38	.49	.32	.21

Let F = Foreign Born

$P(S_4) = .17$

$P(F|S_4) = .32$

WHAT IS THE PROBABILITY THAT A NYC RESIDENT IS FOREIGN BORN?

(INTUITION: BETWEEN .21 & .49)

LAW OF TOTAL PROBABILITY:

$$P(F) = P(F \cap S_1) + P(F \cap S_2) + P(F \cap S_3) + P(F \cap S_4) + P(F \cap S_5)$$

$$= P(S_1)P(F|S_1) + P(S_2)P(F|S_2) + P(S_3)P(F|S_3) + P(S_4)P(F|S_4) + P(S_5)P(F|S_5)$$

$$= (.19)(.29) + (.31)(.38) + (.27)(.49) + (.17)(.32) + (.06)(.21)$$

* ADD UP TO 1

$$= .3722$$

WEIGHTED AVERAGE OF

$$P(F \cap S_i)$$

BAYES' RULE:

RECALL: DEF OF CONDITIONAL PROBABILITY

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B)P(A|B) = P(A \cap B) \quad \text{GENERAL MULT. RULE}$$

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B|A)}{P(B)}$$

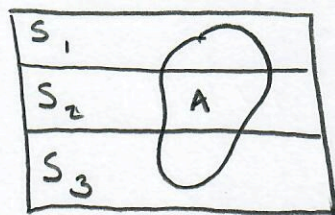
$$\underline{P(A|B)} = \frac{P(A) \underline{P(B|A)}}{P(B)}$$

GIVES ONE CONDITIONAL PROBABILITY

IN TERMS OF ITS OPPOSITE

BAYES' RULE +
LAW OF

BAYES' RULE + LAW OF TOTAL PROB.



$$P(S_i | A) = \frac{P(S_i)P(A|S_i)}{P(A)} \quad (\text{BAYES' RULE})$$

L.O.T.P.

$$P(S_i | A) = \frac{P(S_i)P(A|S_i)}{P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + \dots + P(S_k)P(A|S_k)}$$

ex. THERE IS A TEST FOR A DISEASE :

99.9% OF PEOPLE WITH THE DISEASE TEST POSITIVE.

98% OF PEOPLE WITHOUT THE DISEASE TEST NEGATIVE.

SUPPOSE .5% OF POPULATION HAS THE DISEASE.

WHAT IS THE PROB. THAT A PERSON WHO TESTS POSITIVE HAS THE DISEASE ?

LET : S_1 = SUBPOPULATION THAT HAS THE DISEASE

S_2 = SUBPOPULATION THAT DOES NOT HAVE THE DISEASE

A = TEST POSITIVE FOR THE DISEASE

A^c = TEST NEGATIVE FOR THE DISEASE

NOTATION

GIVEN: $P(A|S_1) = .999$; $P(A^c|S_1) = .001$
"FALSE NEGATIVE"

$P(A^c|S_2) = .98$; $P(A|S_2) = .02$
"FALSE POSITIVE"

$P(S_1) = .005$; $P(S_2) = .995$

FIND $P(S_1|A)$.

BAYES' RULE
 + L.O.T.P.

$$P(S_1|A) = \frac{P(S_1)P(A|S_1)}{P(S_1)P(A|S_1) + P(S_2)P(A|S_2)}$$

$$= \frac{(.005)(.999)}{(.005)(.999) + (.995)(.02)} = .2006$$

$$\approx 20\%$$

Bayes' rule: $P(S_1 | A) = \frac{P(S_1)P(A|S_1)}{P(A)}$

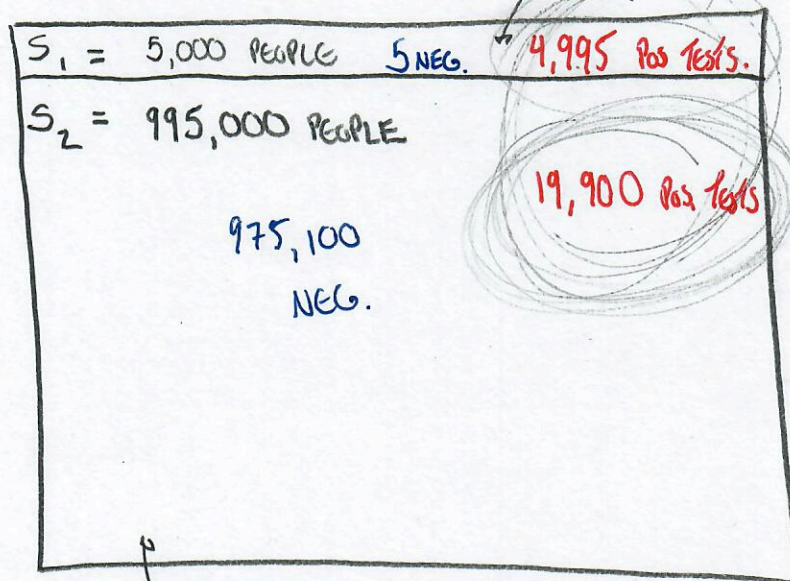
L.O.T.P.

$$P(A) = P(A \cap S_1) + P(A \cap S_2)$$

$$= P(S_1)P(A|S_1) + P(S_2)P(A|S_2)$$

INTUITION:

1,000,000 PEOPLE



99.9% test pos.

2% test pos
98% test neg

$$P(S_1 | A) = \frac{4,995}{4,995 + 19,900}$$

FALSE pos ↑

FALSE NEG ↓

GOLD TEST

= .2006

≈ 20%

OFFICE HOURS

MW 1:30 - 3:30 PM