Name: *ANSWER KEY* 3/27/2017

Math 173 Introduction to Probability and Statistics **Quiz 3**

Please box your final answers. Calculators are allowed, but not required. Answers may be left as fractions and/or expressions that may contain square-root ($\sqrt{\cdot}$), factorial (!), permutation (P_r^n), and combination (C_r^n) notation.

1. City crime records show that 20% of all crimes are violent and 80% are nonviolent, involving theft, forgery, and so on. Ninety percent of violent crimes are reported versus 70% of nonviolent crimes.

   (a) (4 points) What is the overall reporting rate for crimes in the city?

   LET S_1 = EVENT THAT CRIME IS VIOLENT

   S_2 = EVENT THAT CRIME IS NONVIOLENT

   A = EVENT THAT CRIME IS REPORTED

   THEN BY LAW OF TOTAL PROBABILITY, $P(A) = P(S_1)P(A|S_1) + P(S_2)P(A|S_2)$

   $$= (.2)(.9) + (.8)(.7)$$

   $$= \boxed{.74}$$

   (b) (4 points) If a crime in progress is reported to the police, what is the probability that the crime is violent? What is the probability that it is nonviolent?

   BY BAYE'S RULE, $P(S_1|A) = \dfrac{P(S_1)P(A|S_1)}{P(A)}$

   $$= \frac{(.2)(.9)}{.74} = \frac{9}{37} \approx \boxed{.2432}$$

   AND SO $P(S_2|A) = 1 - P(S_1|A) = \boxed{.7568}$

   (c) (2 points) Refer to part (b). If a crime in progress is reported to the police, why is it more likely that it is a nonviolent crime? Wouldnt violent crimes be more likely to be reported? Can you explain these results?

   EVEN THOUGH VIOLENT CRIMES ARE MORE LIKELY TO BE REPORTED,

   ONLY 20% OF CRIMES ARE VIOLENT.

   MOST CRIME IS NON-VIOLENT.

2. A random variable x can equal 0, 1, 2, 3, 4, or 5. A portion of the probability distribution is shown here.

x	0	1	2	3	4	5
$p(x)$	.16	.20	.35	0.10	$p(4)$	.04

(a) (2 points) Find $p(4)$.

$$\sum p(x) = 1 \implies p(4) = \boxed{.15}$$

(b) (4 points) Find the expected value $E[x]$, i.e. the population mean μ.

$$E[x] = \mu = \sum x\, p(x) = (0)(.16) + (1)(.2) + (2)(.35) + (3)(.1) + (4)(.15) + (5)(.04)$$

$$= .2 + .7 + .3 + .6 + .2$$

$$= \boxed{2}$$

(c) (4 points) Find the standard deviation σ for the random variable x.

x	$x - \mu$	$(x-\mu)^2$	$p(x)$
0	-2	4	.16
1	-1	1	.2
2	0	0	.35
3	1	1	.1
4	2	4	.15
5	3	9	.04

$$\sigma^2 = \sum (x-\mu)^2 p(x)$$

$$= (4)(.16) + (1)(.2) + (0)(.35) + (1)(.1) + (4)(.15) + (9)(.04)$$

$$= 1.9$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{1.9} \approx \boxed{1.3784}$$

3. A company has 10 applicants for 3 positions: 3 women and 7 men. Suppose that the 10 applicants are equally qualified and that no preference is given for choosing either gender. Let x equal the number of women chosen to fill the ~~two~~ 3 positions.

(a) (8 points) Fill in the following chart with all possible values of the random variable x, along with the corresponding probabilities $p(x)$.

x	0	1	2	3
$p(x)$	$\frac{35}{120}$	$\frac{63}{120}$	$\frac{21}{120}$	$\frac{1}{120}$
	(.292)	(.525)	(.175)	(.008)

TOTAL # WAYS TO CHOOSE 3 PEOPLE OUT OF 10 = $C^{10}_{3} = \underline{\underline{120}}$

p(0) : # WAYS TO CHOOSE 0 WOMEN & 3 MEN = $C^{3}_{0} \cdot C^{7}_{3} = \underline{\underline{35}}$

SO $p(0) = \frac{35}{120} = \frac{7}{24} \approx .2917$

p(1) : # WAYS TO CHOOSE 1 WOMAN & 2 MEN = $C^{3}_{1} \cdot C^{7}_{2} = \underline{\underline{63}}$

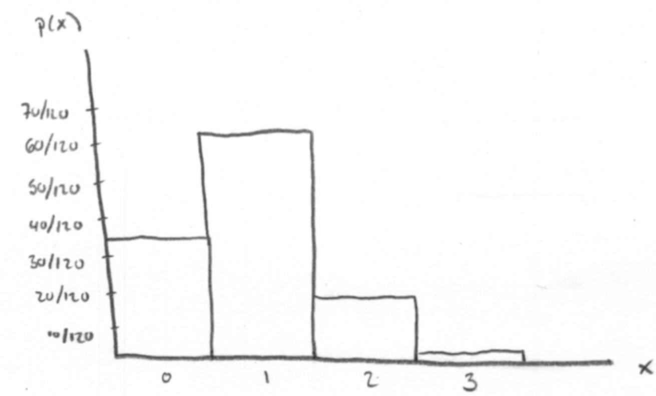
SO $p(1) = \frac{63}{120} = \frac{21}{40} = .525$

p(2) : # WAYS TO CHOOSE 2 WOMEN & 1 MAN = $C^{3}_{2} \cdot C^{7}_{1} = \underline{\underline{21}}$

SO $p(2) = \frac{21}{120} = \frac{7}{40} = .175$

p(3) : # WAYS TO CHOOSE 3 WOMEN & 0 MEN = $C^{3}_{3} \cdot C^{7}_{0} = \underline{\underline{1}}$

(b) (2 points) Sketch a probability histogram for x.
Don't spend much time on this. It doesn't have to be perfect.



4. (4 points) You can insure a \$50,000 diamond for its total value by paying a premium of D dollars. If the probability of loss in a given year is estimated to be .01, what premium should the insurance company charge if it wants the expected gain to equal \$1,000?

LET X = EXPECTED GAIN BY INSURANCE COMPANY

(POSITIVE GAIN IS PROFIT. NEGATIVE GAIN IS LOSS.)

X	$p(x)$
$D - 50{,}000$	$.01$
D	$.99$

$$E[x] = \mu = \sum x\,p(x)$$

$$= (D - 50000)(.01) + (D).99$$

$$= .01D - 500 + .99D$$

$$= D - 500$$

NOW SET $E[x] = 1000$

$$D - 500 = 1000$$

$$\boxed{D = \$1500}$$