Name: * ANSWER KEY * 3/6/2017

Math 173 Introduction to Probability and Statistics **Quiz 2**

Please box your final answers. Calculators are allowed, but not required. Answers may be left as fractions and/or expressions containing factorial (!), permutation (P_r^n), and combination (C_r^n) notation.

1. (a) (4 points) How many distinct 3 letter "words" can you make from 7 distinct letters? Note: In the this question, a word does not have to appear in a dictionary to be considered a "word"; anything with three letters is considered a "word".

$$P_3^7 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = 7 \cdot 6 \cdot 5 = \boxed{210}$$

(b) (4 points) How many ways can you select 3 dogs and 2 cats from a group of 5 dogs and 6 cats if the order is not important?

2 STAGE EVENT : 1ST PICK DOGS 2nd PICK CATS

C_3^5 WAYS × C_2^6 WAYS

$$= \frac{5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{(\cancel{3} \cdot \cancel{2} \cdot \cancel{1})(2 \cdot \cancel{1})} \times \frac{6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{(2 \cdot \cancel{1})(\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1})} = \boxed{150}$$

2. A student prepares for an exam by studying a list of 10 problems. She can solve 6 of them. For the exam, the instructor selects 5 questions at random from the list of 10.

(a) (4 points) What is the probability that the student can solve all 5 problems on the exam?

POSSIBLE EXAMS = C_5^{10}

EXAMS WITH 5 QUESTIONS FROM 6 SHE KNOWS = C_5^6

$$\therefore \text{PROBABILITY} = \frac{C_5^6}{C_5^{10}} = \frac{6}{\frac{10!}{5!5!}} = \frac{6 \cdot 5! 5!}{10!} = \frac{\cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}{\underset{2}{\cancel{10}} \cdot \underset{3}{\cancel{9}} \cdot \cancel{8} \cdot 7 \cdot \cancel{6}} = \boxed{\frac{1}{42}}$$

(b) (4 points) What is the probability that the student can solve exactly 4 problems on the exam?

EXAMS WITH 4 QUESTIONS FROM 6 SHE KNOWS & 1 QUESTION FROM 4 SHE DOESN'T = $C_4^6 \cdot C_1^4$

$$\text{PROBABILITY} = \frac{C_4^6 \cdot C_1^4}{C_5^{10}} = \frac{15 \cdot 4}{252} = \frac{15}{63} = \boxed{\frac{5}{21}}$$

3. An experiment can result in none, one, or both of the events A and B with the probabilities shown in the following table.

	A	A^c	
B	.1	.2	.3
B^c	.3	.4	.7
TOTAL	.4	.6	1

(a) (4 points) Find $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.1}{.3} = \boxed{\frac{1}{3}}$$

(b) (4 points) Find $P(B|A)$.

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{.1}{.4} = \boxed{\frac{1}{4}}$$

(c) (2 points) Are A and B independent events? Explain briefly.

$\boxed{\text{No}}$ BECAUSE (PICK ONE): $P(A|B) \neq P(A)$, $P(B|A) \neq P(B)$, $P(A \cap B) \neq P(A)P(B)$

$\left(\frac{1}{3} \neq .4\right)$. $\left(\frac{1}{4} \neq .3\right)$ $\left(.1 \neq (.4)(.3)\right)$

(d) (2 points) Are A and B mutually exclusive events? Explain briefly.

$\boxed{\text{No}}$ BECAUSE $P(A \cap B) \neq 0$.

4. (6 points) A survey of people in a given region showed that 20% were smokers. The probability of death due to lung cancer, given that a person smoked, was roughly 10 times the probability of death due to lung cancer, given that a person did not smoke. If the probability of death due to lung cancer in the region is .006, what is the probability of death due to lung cancer given that a person is a smoker?

$$P(\text{CANCER}) = P(\text{CANCER} \cap \text{SMOKER}) + P(\text{CANCER} \cap \text{NOT SMOKER})$$

$$= P(\text{SMOKER})P(\text{CANCER}|\text{SMOKER}) + P(\text{NOT SMOKER})P(\text{CANCER}|\text{NOT SMOKER})$$

LET $x = P(\text{CANCER}|\text{SMOKER})$. THEN $.1x = P(\text{CANCER}|\text{NOT SMOKER})$

$$\therefore .006 = .2x + .8(.1x) = .28x$$

$$\Rightarrow x = \frac{.006}{.28} = \boxed{.0214}$$

5. (6 points) Player A has entered a golf tournament but it is not certain whether player B will enter. Player A has probability 1/6 of winning the tournament if player B enters and probability 3/4 of winning if player B does not enter the tournament. If the probability that player B enters is 1/3, find the probability that player A wins the tournament.

LET A = EVENT THAT PLAYER A WINS TOURNAMENT

B = EVENT THAT PLAYER B ENTERS TOURNAMENT

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

$$= P(B)P(A|B) + P(B^c)P(A|B^c)$$

$$= \left(\tfrac{1}{3}\right)\left(\tfrac{1}{6}\right) + \left(1 - \tfrac{1}{3}\right)\left(\tfrac{3}{4}\right)$$

$$= \frac{1}{18} + \frac{1}{2} = \frac{10}{18} = \boxed{\frac{5}{9} \approx .56}$$

Bonus

6. (4 points (bonus)) In a certain lake, the probability of catching a fish is uniform and independent across time. If the probability that you catch at least one fish in an hour is 64%, what is the probability that you catch at least one fish in a half-hour?

LET A = EVENT THAT YOU CATCH 0 FISH IN $\frac{1}{2}$ HOUR.

THEN P(CATCH 0 FISH IN 1 HOUR)

= P(CATCH 0 FISH IN FIRST $\frac{1}{2}$-HOUR AND CATCH 0 FISH IN 2^{nd} $\frac{1}{2}$-HOUR)

= P(A)P(A) BY INDEPENDENCE OF EVENTS

NOTE THAT P(CATCH 0 FISH IN 1 HOUR) = P((CATCH AT LEAST 1 IN 1 HR$)^c$)

= 1 - P(CATCH AT LEAST 1 IN 1 HR) = 1 - .64 = .36

$\therefore P(A)^2 = .36 \Rightarrow P(A) = .6$

SO P(AT LEAST 1 IN HALF-HOUR) = $P(A^c)$ = 1 - .6 = $\boxed{.4}$