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4/28/2017

§6.4 THE NORMAL APPROX. TO THE BINOMIAL PROB. DISTR.

35. (a) YES $\left.\begin{aligned} np &= (25)(.6) = 15 \\ nq &= (25)(.4) = 10 \end{aligned}\right\}$ BOTH > 5 ✓

(b) $\mu = np = \boxed{15}$

$\sigma = \sqrt{npq} = \sqrt{6} \approx \boxed{2.5}$

(c) $P_{BIN.}(x > 9) \approx P_{NORM}(x > 9.5)$

$= P(z > -2.2) = \boxed{.9861}$

36. (a) No.

(b) No.

THE POISSON DISTRIBUTION WOULD BE A GOOD APPROXIMATION IN THIS CASE, BUT THAT IS NOT PART OF OUR SYLLABUS, SO DON'T WORRY ABOUT IT.

37. (a) $\left.\begin{aligned} np &= 7.5 \\ nq &= 17.5 \end{aligned}\right\} > 5 \Rightarrow$ YES ✓

(b) $\mu = np = \boxed{7.5}$

$\sigma = \sqrt{npq} = \boxed{2.29}$

(c) $P_{BIN}(6 \le x \le 9) \approx P_{NORM}(5.5 \le x \le 9.5)$

$= P(-.87 \le z \le .87) = \boxed{.6157}$

38. (a) YES $(np > 5, nq > 5)$

(b) $P_{BIN}(x \geq 6) \approx P_{NORM}(x \geq 5.5) = P(z \geq -1.03) = \boxed{.8485}$

(c) $P_{BIN}(x > 6) \approx P_{NORM}(x \geq 6.5) = P(z \geq -.5164) = \boxed{.6972}$

(d) $P_{BIN}(x \geq 6) = 1 - P(x \leq 5) = 1 - .1509 = \boxed{.8491}$

$P_{BIN}(x > 6) = 1 - P(x \leq 6) = 1 - .3036 = \boxed{.6964}$

CLOSE!

I USED A CALCULATOR, BUT

$$P_{BIN}(x \geq 6) = 1 - \sum_{K=0}^{5} C^{15}_{K} (.5)^{15}$$

$$P_{BIN}(x > 6) = 1 - \sum_{K=0}^{6} C^{15}_{K} (.5)^{15}$$

BECAUSE $p = q = .5$

$\mu = 20$, $\sigma = 4$

39. (a) $P_{BIN}(x > 22) \approx P_{NORM}(x > 22.5) = P(z > .625) = \boxed{.2660}$

(b) $P_{BIN}(x \geq 22) \approx P_{NORM}(x > 21.5) = P(z > .375) = \boxed{.3538}$

(c) $P_{BIN}(20 < x < 25) \approx P_{NORM}(19.5 < x < 25.5)$

$= P(-.125 < z < 1.375) = \boxed{.4651}$

(d) $P_{BIN}(x \leq 25) \approx P_{NORM}(x \leq 25.5) = P(z \leq 1.375) = \boxed{.9154}$

40. (a) $P(4 \le x \le 6) = P(x \le 6) - P(x \le 3)$

$= .780 - .234 = \boxed{.546}$

close!

(b) $\mu = 5$, $\sigma = 2$

$P_{BIN}(4 \le x \le 6) \approx P_{NORM}(3.5 \le x \le 6.5)$

$= P(-.75 \le z \le .75) = \boxed{.5467}$

41. (a) $P(x=5) = P(x \le 5) - P(x \le 4)$

$= .4164 - .2375 = \boxed{.1789}$

(b) $P(x \ge 7) = 1 - P(x \le 6)$

$= 1 - .6080 = \boxed{.3920}$

$\mu = 6$, $\sigma = 2.05$

42. (a) $P_{BIN}(x=5) \approx P_{NORM}(4.5 \le x \le 5.5)$

close

$= P(-.73 \le z \le -.2439) = \boxed{.1710}$

(b) $P_{BIN}(x \ge 7) \approx P_{NORM}(x \ge 6.5)$

$= P(z \ge .2439) = \boxed{.4037}$

43. (a) $P(x \geq 10) = 1 - P(x \leq 9) = \boxed{.2447}$

CLOSE!

(b) $P(x \geq 10) \approx P_{NORM}(x \geq 9.5) = P(z \geq .6849)$

$\mu = 8$ $= \boxed{.2467}$

$\sigma = 2.19$

44. $\mu = 360$ $P(355 \leq x \leq 360) \approx P_{NORM}(354.5 \leq x \leq 360.5)$

$\sigma = 6$

$= P(-2.42 \leq z \leq .08) = \boxed{.5241}$

45. $\mu = np = (50)(.78) = 39$

$\sigma = \sqrt{npq} = 2.93$

(a) $P_{BIN}(x < 30) \approx P_{NORM}(x \leq 29.5) = P(z \leq -3.24) = \boxed{.0001}$

(b) $P_{BIN}(x > 40) \approx P_{NORM}(x \geq 40.5) = P(z \geq .5119) = \boxed{.3044}$

(c) $P_{BIN}(\underbrace{x \leq 39}) \approx P_{NORM}(x \leq 39.5) = P(z \leq .1706) = \boxed{.5677}$

MORE THAN 10 DON'T $\Rightarrow$ LESS THAN 40 DO

46. $\mu = np = (50{,}000)(.001) = 50$

$\sigma = \sqrt{npq} = 7.0675$

$P_{BIN}(x \geq 60) \approx P_{NORM}(x \geq 59.5) = P(z \geq 1.34) = \boxed{.0894}$

I GUESS THAT'S RARE :/

47. $n = 215$

$\mu = (215)(.10) = 21.5$ x = # PEOPLE WHO DO ~~NOT~~ CLAIM THEIR RESERVATION

$\sigma = \sqrt{npq} = 4.3989$

$P(x \geq 15) \approx P_{NORM}(x \geq 14.5) = P(z \geq -1.5913) = \boxed{.9442}$

48. $\mu = (400)(\frac{1}{40}) = 10$

$\sigma = \sqrt{400(\frac{1}{40})(\frac{39}{40})} = 3.1225$

$P(x \geq 19) \approx P_{NORM}(x \geq 18.5) = P(z \geq 2.7222) = \boxed{.0032}$

YES, BECAUSE OTHERWISE YOU WOULD ONLY SEE SUCH A HIGH NUMBER OF PEOPLE WITH LUNG CANCER .3% OF THE TIME (VERY RARE).

49. $n = 49$, $p = .5$ (a) $P(x \geq 26) \approx P_{NORM}(x \geq 25.5) = P(z \geq .2857)$

$\mu = 24.5$ $= \boxed{.3876}$

$\sigma = 3.5$

(b) NO, IT IS FAIRLY LIKELY TO PICK A TALLER CANDIDATE 26 OUT OF 49 TIMES EVEN WHEN $p = .5$.

50. $n = 92$, $p = .15$, $\mu = 13.8$, $\sigma = 3.4249$

(a) $P(x \leq 10) \approx P_{NORM}(x \leq 10.5) = P(z \leq -.9635) = \boxed{.1676}$

(b) $P(15 \leq x \leq 20) \approx P_{NORM}(14.5 \leq x \leq 20.5) = P(.2044 \leq z \leq 1.9563)$

$= \boxed{.3938}$

(c) MORE THAN 80 RH - POSITIVE

$\Leftrightarrow$ LESS THAN 12 RH - NEGATIVE

$$P(x < 12) \approx P_{NORM}(x \le 11.5) = P(z \le -.6716) = \boxed{.2569}$$

51. $n = 500 \quad p = .25$

$\mu = 125 \quad \sigma = 9.6825$

(a) $P(x = 150) \approx P_{NORM}(149.5 \le x \le 150.5) = P(2.5303 \le z \le 2.6336)$

$$= \boxed{.0015}$$

(b) $P(120 \le x \le 150) \approx P_{NORM}(119.5 \le x \le 150.5) = P(-.5680 \le z \le 2.6336)$

$$= \boxed{.7108}$$

(c) $P(x < 150) \approx P_{NORM}(x \le 149.5) = P(z \le 2.5303) = \boxed{.9943}$

(d) $P(x \ge 232) \approx P_{NORM}(x \ge 231.5) = P(z \ge 10.9992) =$

$$.\underbrace{000000000000000000000000000}_{27\ 0\text{'s}!}2$$

> YES, THAT WOULD BE UNUSUAL.
>
> I WOULD CONCLUDE THAT p IS SIGNIFICANTLY HIGHER THAN .25

52. PRESSURE TO WORK TOO MUCH : $\mu = (25)(.60) = 15$

$\sigma = 2.4495$

WISH FOR MORE FAMILY TIME : $\mu = (25)(.8) = 20$

$\sigma = 2$

(a) $P(x > 20) \approx P_{NORM}(x \geq 20.5) = P(z \geq 2.2454) = \boxed{.0124}$

(b) $P(x > 20) = 1 - P(x \leq 20) = 1 - .9905 = \boxed{.0095}$

(c) $P(15 \leq x \leq 20) = P(x \leq 20) - P(x \leq 14)$

$= .5793 - .0056 = \boxed{.5737}$

CLOSE!

(d) $P(15 \leq x \leq 20) \approx P_{NORM}(14.5 \leq x \leq 20.5)$

$= P(-2.75 \leq z \leq .25) = \boxed{.5957}$

53. $n = 50$, $p = .62$, $\mu = 31$, $\sigma = 3.4322$

(a) $\mu = np = \boxed{31}$

(b) $\sigma = \sqrt{npq} = \boxed{3.4322}$

(c) $P(x \leq 25) \approx P_{NORM}(x \leq 25.5) = P(z \leq -1.6025)$

$= \boxed{.0545}$

YES, PRETTY UNUSUAL.

(I GUESS :-))