John Adamski
4/20/2017

§6.3 Tabulated Areas & the Normal Probability Distribution

1. (a) $P(z < 2) = \boxed{.9772}$

(b) $P(z > 1.16) = 1 - P(z < 1.16)$

$= 1 - .8770 = \boxed{.1230}$

(c) $P(-2.33 < z < 2.33) = P(z < 2.33) - P(z < -2.33$

$= .9901 - .0099 = \boxed{.9802}$

(d) $P(z < 1.88) = \boxed{.9699}$

2. (a) $P(z > 5) = 1 - P(z < 5) = 1 - .9999\ldots \approx \boxed{0}$

(b) $P(-3 < z < 3) = P(z < 3) - P(z < -3)$

$= .9987 - .0013 = \boxed{.9974}$

(c) $P(z < 2.81) = \boxed{.9975}$

(d) $P(z > 2.81) = 1 - P(z < 2.81) = 1 - .9975 = \boxed{.0025}$

3. (a) $P(z < 1.6) = \boxed{.9452}$

(b) $P(z < 1.83) = \boxed{.9664}$

(c) $P(z < .90) = \boxed{.8159}$

(d) $P(z < 4.18) \approx \boxed{1}$

4. $P(-1.4 < z < 1.4) = P(z < 1.4) - P(z < -1.4)$

$= .9192 - .0808 = \boxed{.8384}$

$P(-3 < z < 3) = P(z < 3) - P(z < -3)$

$= .9987 - .0013 = \boxed{.9974}$

5. (a) $P(-1.43 < z < .68) = P(z < .68) - P(z < -1.43)$

$= .7517 - .0764 = \boxed{.6753}$

(b) $P(.58 < z < 1.74) = P(z < 1.74) - P(z < .58)$

$= .9591 - .7190 = \boxed{.2401}$

(c) $P(-1.55 < z < -.44) = P(z < -.44) - P(z < -1.55)$

$= .3300 - .0606 = \boxed{.2694}$

(d) $P(z > 1.34) = 1 - P(z < 1.34) = 1 - .9099 = \boxed{.0901}$

(e) $P(z < -4.32) \approx \boxed{0}$

6. (a) $P(z < 2.33) = \boxed{.9901}$

(b) $P(z < 1.645) \approx$ AVERAGE OF $P(z < 1.64)$ & $P(z < 1.65)$

$$\approx \frac{P(z < 1.64) + P(z < 1.65)}{2} = \frac{.9495 + .9505}{2}$$

$$= \boxed{.95}$$

(c) $P(z > 1.96) = 1 - P(z < 1.96) = 1 - .9750 = \boxed{.0250}$

(d) $P(-2.58 < z < 2.58) = P(z < 2.58) - P(z < -2.58)$

$$= .9951 - .0049 = \boxed{.9902}$$

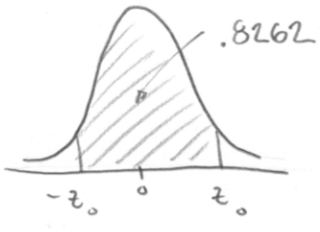
7. (a) $1 - .025 = .975$

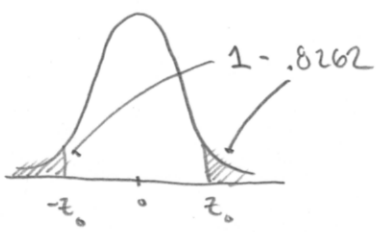
FIND z_0 IN TABLE 3 SUCH THAT $P(z < z_0) = .975$

$\boxed{z_0 = 1.96}$

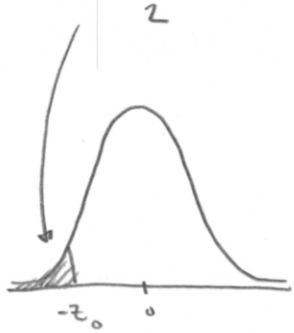
(b) FROM TABLE 3: $\boxed{1.44}$

8.





$\frac{1 - .8262}{2} = .0869$

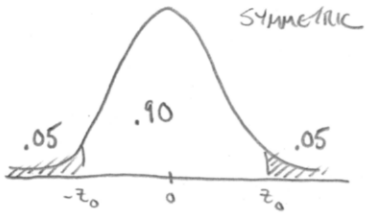


$-z_0 = -1.36 \rightarrow \boxed{z_0 = 1.36}$

9. (a) $z_0 = 1.65$

(b) $z_0 = \boxed{-1.645}$ (BETWEEN -1.64 & -1.65)

10. (a)



$P(z < z_0) = .95$

$\boxed{z_0 = 1.645}$

(b) $P(z < z_0) = .995 \rightarrow \boxed{z_0 = 2.575}$

11. (a) $P(z < z_0) = .9 \rightarrow z_0 = \boxed{1.28}$

(b) $P(z < z_0) = .95 \rightarrow z_0 = \boxed{1.645}$

(c) $P(z < z_0) = .98 \rightarrow z_0 = \boxed{2.055}$

(d) $P(z < z_0) = .99 \rightarrow z_0 = \boxed{2.33}$

12. CONVERT $z = \dfrac{x - \mu}{\sigma}$

(a) $\dfrac{13.5 - 10}{2} = 1.75$

$\therefore P(x > 13.5) = P(z > 1.75) = \boxed{.0401}$

(b) $\dfrac{8.2 - 10}{2} = -.9$

$\therefore P(x < 8.2) = P(z < -.9) = \boxed{.1841}$

(c) $\frac{9.4 - 10}{2} = -.3$, $\frac{10.6 - 10}{2} = .3$

$P(9.4 < x < 10.6) = P(-.3 < z < .3) = \boxed{.2358}$

13. (a) $\frac{1.00 - 1.2}{.15} = -1.33$ $\frac{1.10 - 1.2}{.15} = -.67$

$P(1.00 < x < 1.10) = P(-1.33 < z < -.67) = \boxed{.1597}$

(b) $\frac{1.38 - 1.2}{.15} = 1.2$

$P(x > 1.38) = P(z > 1.2) = \boxed{.4207}$

(c) $\frac{1.35 - 1.2}{.15} = 1$ $\frac{1.5 - 1.2}{.15} = 2$

$P(1.35 < x < 1.5) = P(1 < z < 2) = \boxed{.1359}$

14. $P(z > z_0) = .8023 \rightarrow z_0 = .85$ (FROM TABLE 3)

$z_0 = \frac{x_0 - \mu}{\sigma} \rightarrow .85 = \frac{7.5 - \mu}{2}$

$1.7 = 7.5 - \mu \rightarrow \boxed{\mu = 5.8}$

15. $P(x > x_0) = .01 \longrightarrow P(x < x_0) = .99$

FROM TABLE 3, $P(z < z_0) = .99 \Rightarrow z_0 = 2.33$

$$z_0 = \frac{x_0 - \mu}{\sigma} \longrightarrow 2.33 = \frac{x_0 - 35}{10}$$

$$23.3 = x_0 - 35 \longrightarrow \boxed{x_0 = 58.3}$$

16. $x = 0 \longrightarrow z = \dfrac{0 - 50}{15} = -3.33$

$P(x \leq 0) = P(z \leq -3.33) \approx 0$ SO YES, IT WOULD BE UNUSUAL

17. $P(z > z_1) = .9772 \Rightarrow P(z < z_1) = .0228$

$\Rightarrow z_1 = -2$ (FROM TABLE 3)

$P(z > z_2) = .9332 \Rightarrow P(z < z_2) = .0668$

$\Rightarrow z_2 = -1.5$

z_1 CORRESPONDS TO $x_1 = 4 \longrightarrow -2 = \dfrac{4 - \mu}{\sigma}$ OR $\mu = 4 + 2\sigma$

z_2 CORRESPONDS TO $x_2 = 5 \longrightarrow -1.5 = \dfrac{5 - \mu}{\sigma}$ OR $\mu = 5 + 1.5\sigma$

$\therefore 4 + 2\sigma = 5 + 1.5\sigma \longrightarrow .5\sigma = 1 \longrightarrow \boxed{\sigma = 2}$

THEN $\mu = 4 + 2(2) = 8 \longrightarrow \boxed{\mu = 8}$

18. (a) $P(x > 1) = P(z > 0) = \boxed{.5}$

(b) $P(.95 < x < 1.05) = P(-.33 < z < .33) = \boxed{.2586}$

$\frac{.95 - 1}{.15} = -.33$ $\quad$ $\frac{1.05 - 1}{.15} = .33$

(c) $P(x < .8) = P(z < -1.33) = \boxed{.0918}$

(d) $P(x \geq 1.45) = P(z \geq 3) = .0013$ YES, UNUSUAL

BUT IT HAPPENS 13 TIMES IN 1000

19. (a) $P(x > 72) = P(z > .71) = \boxed{.2389}$

(b) $P(68 < x < 73) = P(-.4286 < z < 1) = \boxed{.5072}$

(c) $P(x \geq 73) = P(z > 1) = .1587 \approx \frac{1}{6}$

NOT THAT UNUSUAL

(d) $\frac{18}{43} = .4186$, HIGHER THAN PROPORTION OF GENERAL POPULATION .2389

20. (a) $P(3 < x < 5) = P(-.67 < z < .67) = \boxed{.4971}$

(b) $P(x < 3) = P(z < -.67) = \boxed{.2514}$

(c) $P(x > 6) = P(z > 1.33) = \boxed{.0918}$

21. (a) $P(60 < x < 80) = P(-.87 < z < .38) = \boxed{.4559}$

(b) $P(x > 100) = P(z > 1.63) = \boxed{.0516}$

(c) $P(x < 40) = P(z < -2.12) = \boxed{.0170}$

22. (a) $P(x < 40) = P(z < -1.25) = \boxed{.1056}$

$P(x < 50) = P(z < 0) = \boxed{.5}$

(b) $P(x < 60) = P(z < 1.25) = \boxed{.8944}$

23. $P(x > 1300) = P(z > 1.01) = \boxed{.1562}$

$P(x > 1500) = P(z > 3.03) = \boxed{.0012}$

24. $P(x > 50) = P(z > 1.64) = \boxed{.0505}$

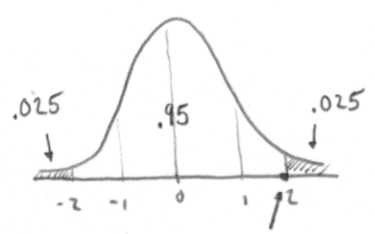
29. (a) $P(x > 40) = P(z > 1.67) = \boxed{.0475}$

(b) ASSUME ONE SUNFLOWER HAS NO INFLUENCE ON THE OTHER. THAT IS, THESE ARE INDEPENDENT EVENTS.

So $P(1^{st}\ \text{FLOWER} > 40 \cap 2^{nd}\ \text{FLOWER} > 40)$

$= P(x > 40) \cdot P(x > 40) = (.0475)^2 = \boxed{.0023}$

(c)



$P(z < z_0) = .975$

$z_0 = 1.96$

(FROM TABLE)

$\Rightarrow P(-1.96 < z < 1.96) = .95$

$x = \mu - 1.96\mu$ $\quad$ $x = \mu + 1.96\sigma$

$= 35 - 1.96(3)$ $\quad$ $= 35 + 1.96(3)$

$= 29.12$ $\quad$ $= 40.88$

(d) $P(z < z_0) = .90 \Rightarrow z_0 = 1.28$ (FROM TABLE)

$\Rightarrow x = \mu + 1.28\sigma = 35 + 1.28(3) = \boxed{38.84}$

26. $P(x > 22) = P(z > 1.5) = \boxed{.0668}$

27. $P(x > 4.25) = P(z > -2.5) = \boxed{.9938}$

$P(x < 4.375) = P(z < -1.87) = \boxed{.0307}$

28. (a) $P(x > 2.5) = P(z > 2.11) = \boxed{.0174}$

(b) $P(x < 1) = P(z < -1.22) = \boxed{.1112}$

29. $P(x > 100) = P(z > 1.67) = \boxed{.0475}$

30. $P(z > z_0) = .01$ (1 IN 100)

$P(z < z_0) = .99 \Rightarrow z_0 = 2.33$ ← 99% OF DATA LIE BELOW

$x = \mu + 2.33\sigma = 2000$

$\mu + 2.33(25.7) = 2000$

$\boxed{\mu = 1940.119}$

31. $P(z < z_0) = .95 \Rightarrow z_0 = 1.645$ (FROM TABLE 3)

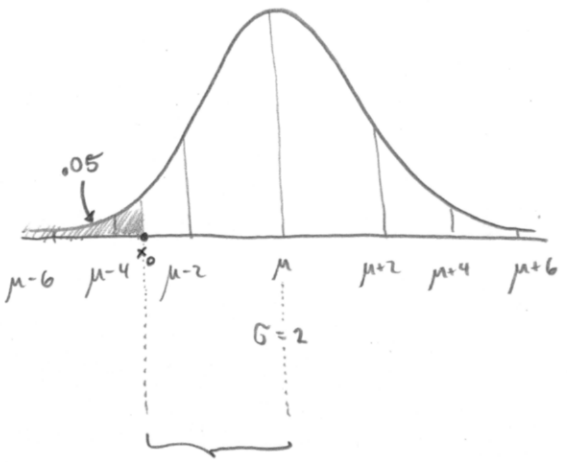
$x_0 = \mu + 1.645\,\sigma = 100{,}000$

$\mu + 1.645(10{,}000) = 100{,}000$

$\mu = 83{,}550$ ← AND THIS IS "20,000 WORDS IN EXCESS OF CONTRACT"

$\therefore$ **CONTRACT = 63,550**

32.



$P(z < z_0) = .05$

$z_0 = -1.645$

x_0 IS $1.645\,\sigma$ BELOW μ

μ IS $1.645\,\sigma$ ABOVE x_0 = $1.645(2)$ ABOVE x_0

= **3.29 LBS/in² ABOVE THE CUSTOMER'S SPECIFIED TENSION**

33. (a) $P(x > 95) = P(z > .5) =$ **.3085**

(b) $P(95 < x < 115) = P(.5 < z < 1.5) =$ **.2417**

(c) $P(x > 115)^2 = P(z > 1.5)^2 = (.0668)^2 =$ **.0045**

34. (a) $P(60 < x < 100) = P(-1.5 < z < 1.83) =$ **.8996**

(b) $P(z < z_0) = .95 \Rightarrow z_0 = 1.645$

$\rightarrow x_0 = \mu + 1.645\sigma = 78 + 1.645(12) =$ **97.74**

(c) $P(x \geq 110) = P(z \geq 2.67) =$ **.0038**

↑ YES, THAT'S UNUSUAL