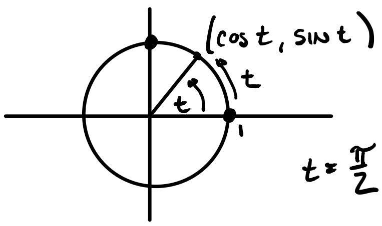
$$f(x) = \frac{2\cos(x)}{1 - \sin x}$$

"DENOM $\neq 0$"

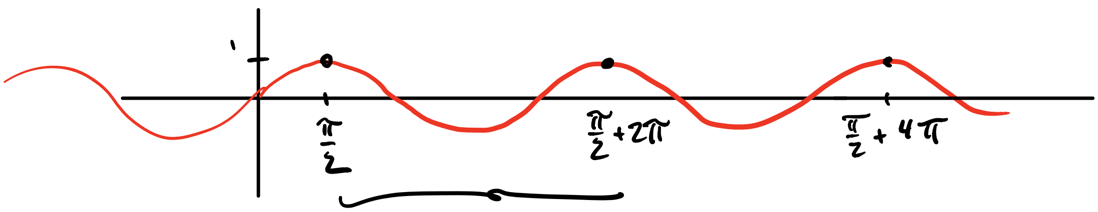
$$\text{Dom}(f) = \{x \mid 1 - \sin x \neq 0\} = \{x \mid x \neq \tfrac{\pi}{2} \pm n2\pi,\ n = 0, \pm 1, \pm 2, \ldots\}$$

ALL X EXCEPT $1 - \sin x = 0$

$\sin x = 1$



$t = \frac{\pi}{2} \pm n2\pi$



$$f(x) = \frac{1}{1 + ax}$$

$$f \circ f(x) = f(f(x)) = \frac{1}{1 + a f(x)}$$

$$= \underbrace{\frac{1}{1 + a\left(\frac{1}{\underbrace{1 + ax}_{(1)}}\right)}}_{(2)}$$

FIND Dom$(f \circ f)$

(1) $1 + ax \neq 0$

$x \neq -\frac{1}{a}$

(2) $1 + a\left(\frac{1}{1 + ax}\right) \neq 0$

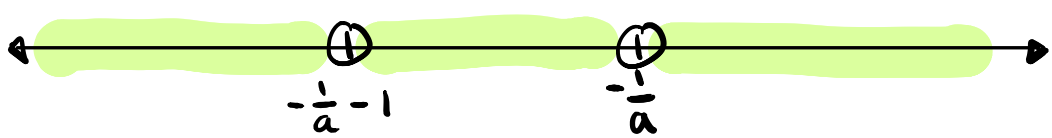
$$\frac{1}{1+ax} \neq -\frac{1}{a}$$

$$-a \neq 1+ax$$

$$-\frac{a}{a} - \frac{1}{a} = \frac{-a-1}{a} \neq x$$

$$-1 - \frac{1}{a} =$$

DOMAIN: $x \neq -1 - \frac{1}{a}, -\frac{1}{a}$



$$\left(-\infty, -\frac{1}{a} - 1\right) \cup \left(-1, -\frac{1}{a}\right) \cup \left(-\frac{1}{a}, \infty\right)$$

SYMBOL: CALL PAD UNDER SET NOTATION

NOTE: $f \circ f(x) = \frac{1}{1 + a\left(\frac{1}{1+ax}\right)} \cdot \frac{(1+ax)}{(1+ax)}$

ASSUMING $1+ax \neq 0$

$\frac{1}{2} = \frac{2}{4}$

$\frac{1}{2} \cdot \frac{A}{A} = \frac{A}{2A}$

$A \neq 0$

$$= \underbrace{\frac{1+ax}{1+ax+a}}$$

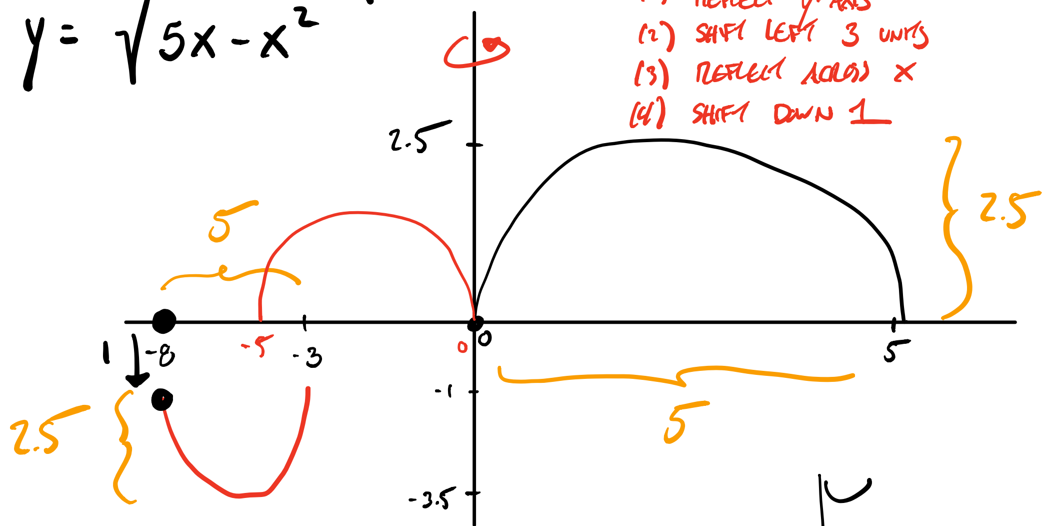
DIFFERENT. (DIFF. DOMAINS)

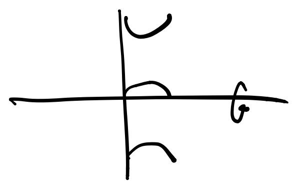
Dom: $1 + ax + a \neq 0$

$$x \neq \frac{-1-a}{a} = -\frac{1}{a} - 1$$

$y = \sqrt{5x - x^2}$

(1) REFLECT Y-AXIS
(2) SHIFT LEFT 3 UNITS
(3) REFLECT ACROSS X
(4) SHIFT DOWN 1





- SHIFT LEFT 8
- REFLECT ACROSS X-AXIS
- SHIFT DOWN 1 UNIT

$y = f(x) \rightarrow y = f(x+8)$ $\qquad f(x) = \sqrt{5x - x^2}$

$\rightarrow y = -f(x+8)$

$\rightarrow y = -f(x+8) - 1$

$$\boxed{y = -\sqrt{5(x+8) - (x+8)^2} - 1}$$

$5x + 40 - (x^2 + 16x + 64)$

$$-\sqrt{-x^2 - 11x - 24} - 1$$

$$f(x) = \sqrt{5x - x^2}$$

(1) REFLECT Y-AXIS
(2) SHIFT LEFT 3 UNITS
(3) REFLECT ACROSS X
(4) SHIFT DOWN 1

$$y = f(x) \xrightarrow{(1)} y = f(-x) \xrightarrow{(2)} y = f(-(x+3))$$

$$\xrightarrow{(3)} y = -f(-(x+3))$$

$$\xrightarrow{(4)} y = -f(-(x+3)) - 1$$

$$y = -\sqrt{5(-(x+3)) - (-(x+3))^2} - 1$$

$$5(-x-3)$$

$$-5x - 15 - (x^2 + 6x + 9)$$

$$-\sqrt{-x^2 - 11x - 24} - 1$$

§1.6 Calculating Limits Using Limit Laws

Def: $\lim_{x \to a} f(x) = L$ if & only if

the output $f(x)$ approaches L $(f(x) \to L)$

as the input x approaches a $(x \to a)$.

$(x \neq a)$

Limit Laws:

Suppose $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist.

Let c be a constant.

If these limits do not exist then this eq is not true.

1. $\lim_{x \to a} \left(f(x) \pm g(x) \right) = \left(\lim_{x \to a} f(x) \right) \pm \left(\lim_{x \to a} g(x) \right)$

"The limit of the sum/difference is the sum/difference of the limits"

(provided that they exist)

Why? Say $f(x) \to L$ as $x \to a$

$g(x) \to M$ as $x \to a$

Then $\left(f(x) \pm g(x) \right) \to L \pm M$ as $x \to a$

2. $$\lim_{x\to a}\left(c f(x)\right) = c\left(\lim_{x\to a} f(x)\right)$$

IF $f(x) \to L$ AS $x \to a$
THEN $c f(x) \to cL$ AS $x \to a$

3. $$\lim_{x\to a}\left(f(x)g(x)\right) = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right)$$

$$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$$

← ASSUMING $\lim_{x\to a} g(x) \neq 0$

"THE LIMIT OF THE PRODUCT/QUOTIENT IS THE PRODUCT/QUOTIENT OF THE LIMITS"

4. $$\lim_{x\to a}\left(f(x)^m\right) = \left(\lim_{x\to a} f(x)\right)^m$$

FOR $m = 1, 2, 3, \ldots$

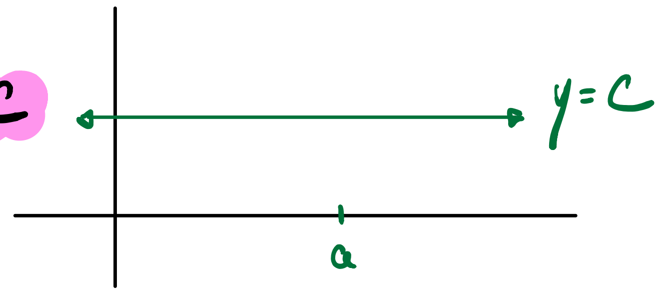
IF $f(x) \to L$ AS $x \to a$
THEN $f(x)^m \to L^m$ AS $x \to a$

$$\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$$

IF $f(x) \to L$ AS $x \to a$
THEN $\sqrt[n]{f(x)} \to \sqrt[n]{L}$ AS $x \to a$

Special Limits

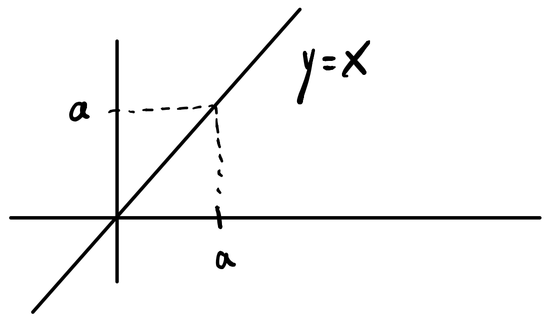
(1) $\displaystyle\lim_{x \to a} c = c$



$c \to c$ as $x \to a$ ✓

(2) $\displaystyle\lim_{x \to a} x = a$

input = output



$x \to a$ as $x \to a$

Combining Limit Laws + Special Limits

__ex.__ Let $f(x) = 3x^4 - 2\sqrt{x} + 7$

Find $\displaystyle\lim_{x \to 2} f(x)$.

$$\lim_{x \to 2} \left(3x^4 - 2\sqrt{x} + 7\right)$$

LL1 $\displaystyle = \lim_{x \to 2} 3x^4 - \lim_{x \to 2} 2\sqrt{x} + \lim_{x \to 2} 7$

LL2 $\displaystyle = 3\lim_{x \to 2} x^4 - 2\lim_{x \to 2} \sqrt{x} + \lim_{x \to 2} 7$

$$\overset{LL4}{=} 3\left(\lim_{x\to 2} x\right)^4 - 2\sqrt{\lim_{x\to 2} x} + \lim_{x\to 2} 7$$

$$\downarrow (2) \qquad \downarrow (2) \qquad \downarrow (1)$$

$$= 3(2)^4 - 2\sqrt{(2)} + (7)$$

$$= 48 - 2\sqrt{2} + 7 = \boxed{55 - 2\sqrt{2}}$$

ex. FIND $\displaystyle\lim_{x\to 2} \frac{3x^4 - 2\sqrt{x} + 7}{3x + 1}$

SUBSTITUTE $x = 2$

$$\overset{LL3}{=} \frac{\displaystyle\lim_{x\to 2} 3x^4 - 2\sqrt{x} + 7}{\displaystyle\lim_{x\to 2} (3x+1)}$$

$$= \frac{55 - 2\sqrt{2}}{3\displaystyle\lim_{x\to 2} x + \lim_{x\to 2} 1} = \boxed{\frac{55 - 2\sqrt{2}}{7}}$$

$$3(2) + 1$$

Def: AN ALGEBRAIC FUNCTION IS ANY FUNCTION THAT IS "BUILT" FROM CONSTANT MULTIPLES, SUMS/DIFFERENCES, PRODUCTS/QUOTIENTS

OF POWER / ROOT FUNCTIONS.

ex. $f(x) = \dfrac{7x^3 + 4x^4\sqrt[3]{x} + 1}{x^2\sqrt{3x^2+1}}$

DIRECT SUBSTITUTION RULE

IF f IS AN ALGEBRAIC FUNCTION
AND IF a IS IN $\mathrm{Dom}(f)$
THEN $\lim_{x \to a} f(x) = f(a)$

CAN JUST SUBSTITUTE $x = a$.

NOTE: THIS THE DEFINITION OF
"f IS CONTINUOUS AT $x = a$"
SO D.S.R. SAYS "ALGEBRAIC FUNCTIONS
ARE CONTINUOUS ON THEIR DOMAINS"
(CONTINUITY WILL BE DISCUSSED IN §1.8)

ex. $\lim_{x \to 2} \underbrace{\dfrac{x-1}{x^2+2x-3}}_{f(x)} = \dfrac{(2)-1}{(2)^2+2(2)-3} = \boxed{\dfrac{1}{5}}$

IF $f(2)$ IS DEFINED THEN $\lim_{x \to 2} f(x) = f(2)$

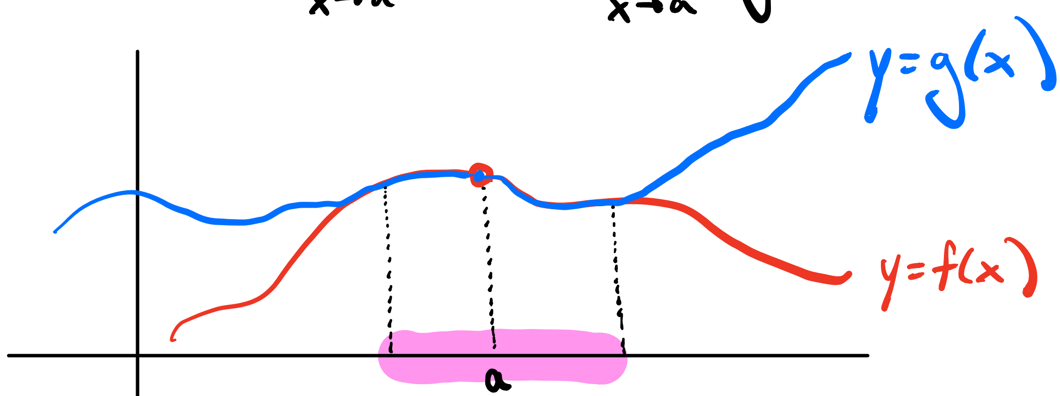
ex. $$\lim_{x \to 1} \frac{x-1}{x^2+2x-3}$$

NOTE: $f(1) = \frac{0}{0}$ UNDEFINED.

SO D.S.R. DOES NOT APPLY:
1 NOT IN DOM(f).

THEOREM: IF $f(x) = g(x)$ FOR ALL x NEAR a,
EXCEPT FOR $x = a$,

THEN $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$.



$f(x) = g(x)$ $\forall x$ NEAR a.
"FOR ALL"

ex. $$\lim_{t \to -3} \frac{t^2-9}{2t^2+7t+3}$$

SUBSTITUTE: $\frac{(-3)^2-9}{2(-3)^2+7(-3)+3} = \frac{0}{0}$ $\times$

D.S.P. DOES NOT APPLY
$-3 \notin$ DOM(f)

NOTE: $\dfrac{t^2-9}{2t^2+7t+3} = \dfrac{\cancel{(t+3)}(t-3)}{(2t+1)\cancel{(t+3)}}$

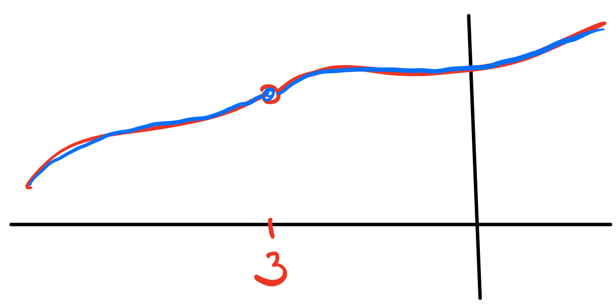
$= \dfrac{t-3}{2t+1}$ $\forall x$ EXCEPT $x=-3$

D.S.R.

$\lim_{x\to -3} \dfrac{t^2-9}{2t^2+7t+3} = \lim_{x\to -3} \dfrac{t-3}{2t+1} = \dfrac{(-3)-3}{2(-3)+1}$

$(x \neq -3)$

$= \dfrac{-6}{-5} = \boxed{1.2}$



HW §1.4, 1.5 POSTED

DUE 9/15.

NOTABILITY $\approx$ \$8