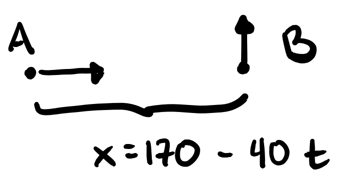
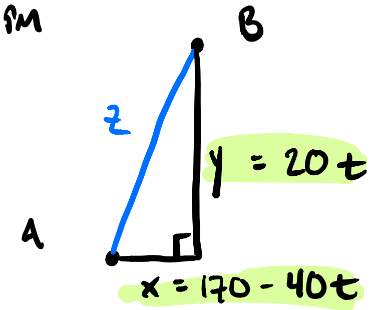
At noon, ship A is 170 km west of ship B. Ship A is sailing east at 40 km/h and ship B is sailing north at 20 km/h. How fast is the distance between the ships changing at 4:00 PM?

$\boxed{\dfrac{120}{\sqrt{65}}}$ km/h

NOON



4 PM



t HOURS

RELATE QUANTITIES: $x^2 + y^2 = z^2 \longrightarrow z = \sqrt{x^2 + y^2}$

RELATE RATES: $\cancel{2}x \dfrac{dx}{dt} + \cancel{2}y \dfrac{dy}{dt} = \cancel{2}z \dfrac{dz}{dt}$

FIND $\dfrac{dz}{dt}$ AT 4 PM : $(t = 4)$ WHEN $x = 10$ $\quad \dfrac{dx}{dt} = -40$

$y = 80 \quad \dfrac{dy}{dt} = 20$

$$\frac{dz}{dt} = \frac{x \dfrac{dx}{dt} + y \dfrac{dy}{dt}}{z}$$

$$\left.\frac{dz}{dt}\right|_{4PM} = \frac{(10)(-40) + (80)(20)}{\sqrt{10^2 + 80^2}} = \frac{1200}{\sqrt{6500}} = \frac{120\cancel{0}}{\cancel{10}\sqrt{65}}$$

$$= \frac{120}{\sqrt{65}}$$

§3.1 Maximum & Minimum Values

- Definitions: Absolute vs. Local
- Extreme Value Theorem
- Fermat's Theorem (Do's & Don'ts)
- Critical Numbers
- The Closed Interval Method

Def: A point $c \in \text{Dom}(f)$ is an absolute maximum of f if

$$f(c) \geq f(x) \quad \forall x \in \text{Dom}(f),$$

and we call $f(c)$ the absolute maximum value.

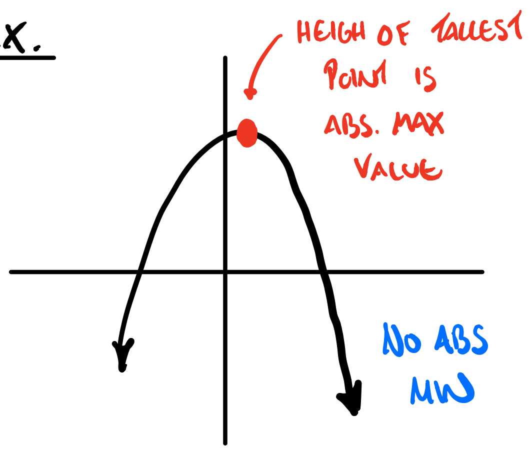
A point $c \in \text{Dom}(f)$ is an absolute minimum of f if

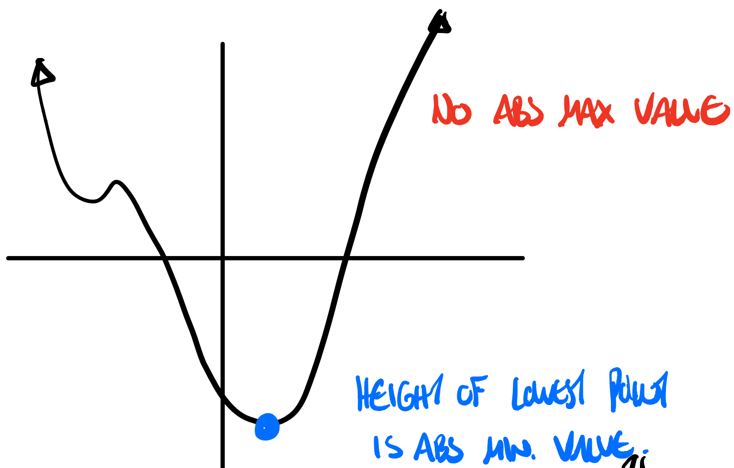
$$f(c) \leq f(x) \quad \forall x \in \text{Dom}(f),$$

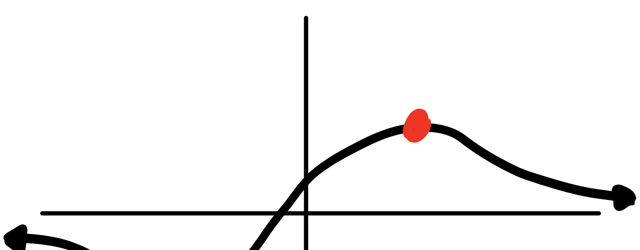
and we call $f(c)$ the absolute minimum value.

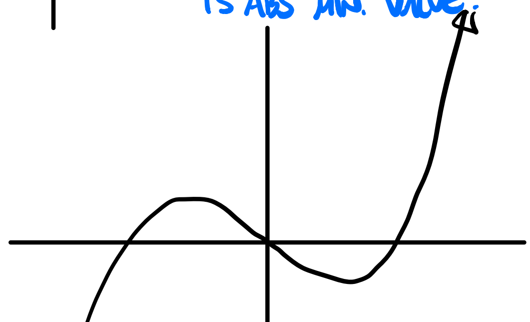
"Value" refers to output.

ex.



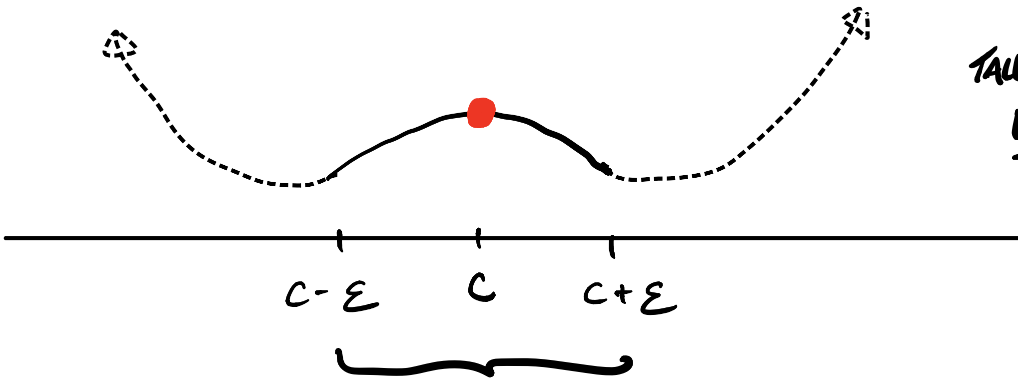






NO ABS MAX OR MIN.

<u>Def</u>: A POINT $c \in \mathrm{Dom}(f)$ IS A LOCAL MAXIMUM OF f IF $\exists\, \varepsilon > 0$ s.t. $f(c) \geq f(x)$ $\forall x \in (c-\varepsilon, c+\varepsilon)$, AND WE CALL $f(c)$ A LOCAL MAXIMUM VALUE.

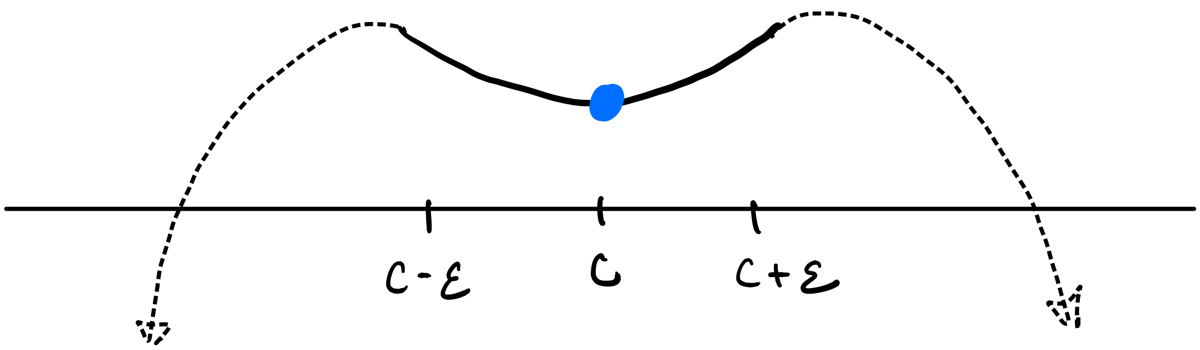


TALLEST POINT LOCALLY

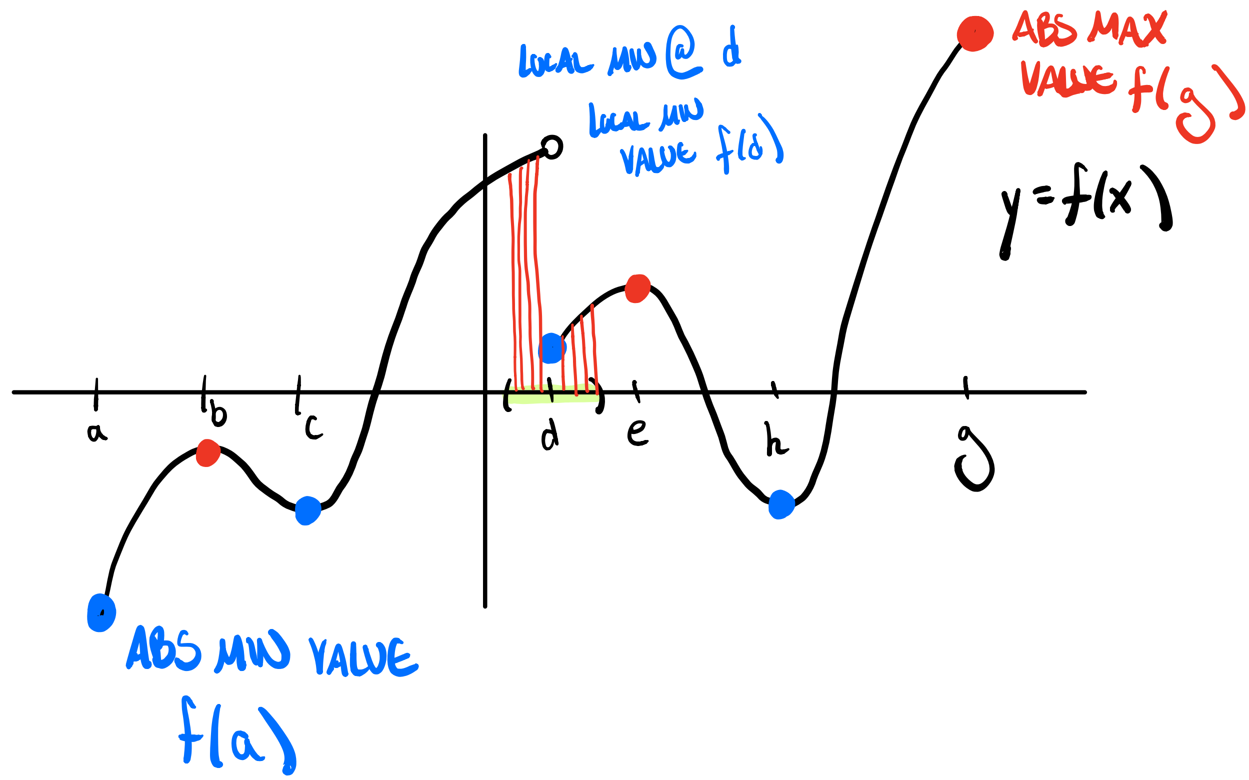
OPEN INTERVAL CENTERED AT c

A LITTLE TO THE LEFT & A LITTLE TO THE RIGHT

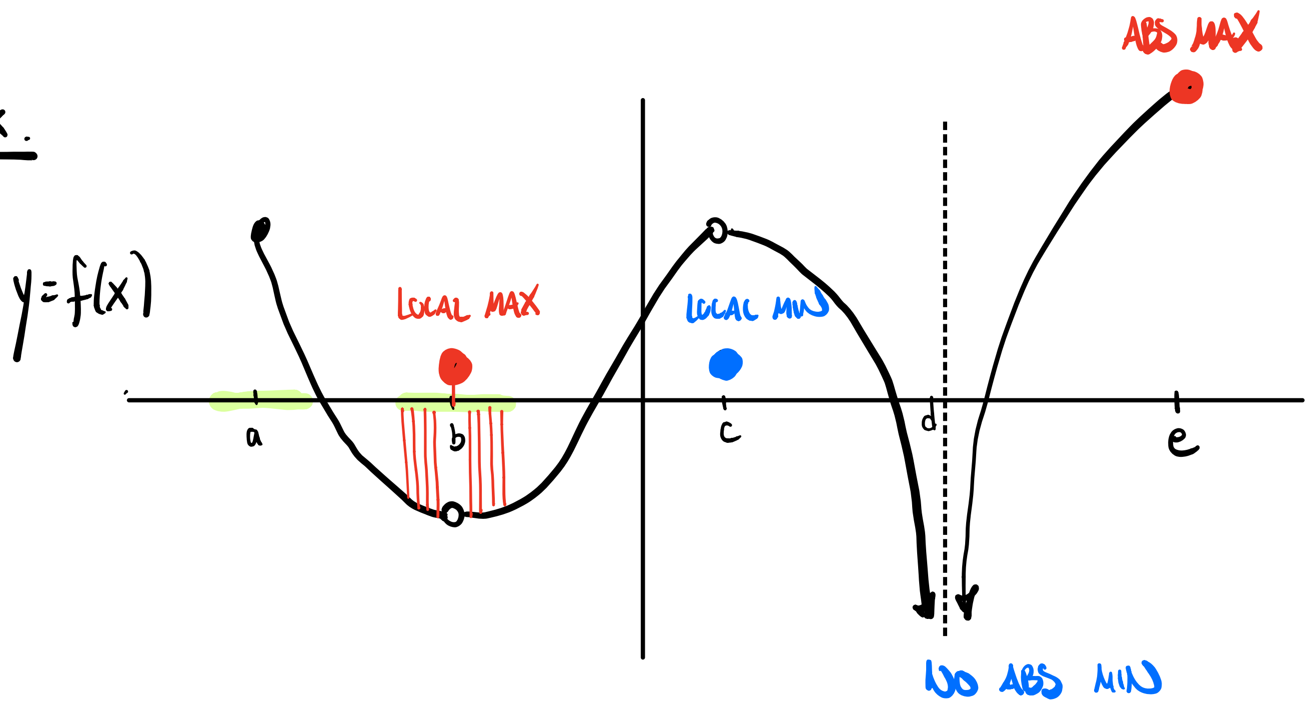
A POINT $c \in \mathrm{Dom}(f)$ IS A LOCAL MINIMUM OF f IF $\exists\, \varepsilon > 0$ s.t. $f(c) \leq f(x)$ $\forall x \in (c-\varepsilon, c+\varepsilon)$, AND WE CALL $f(c)$ A LOCAL MINIMUM VALUE.



ex.



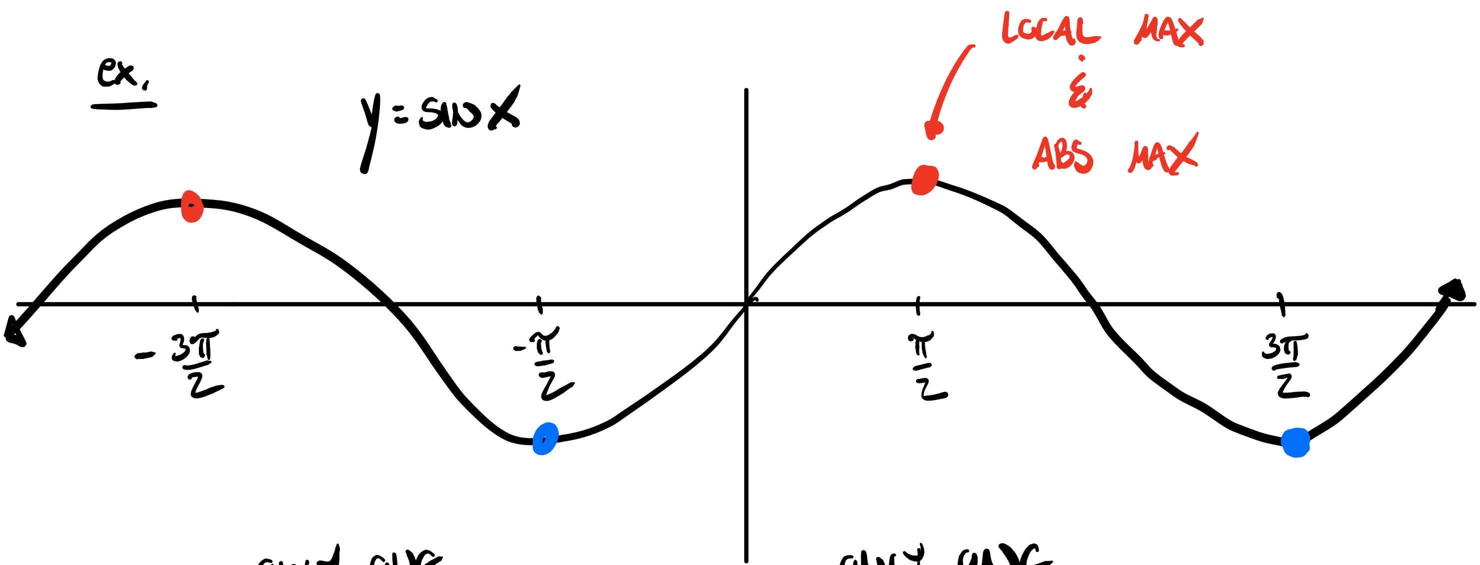
ex.



NOTE: ENDPOINTS OF THE DOMAIN CANNOT BE LOCAL MAX'S OR MIN'S.

(THEY CAN BE ABSOLUTE MAX'S & MIN'S)

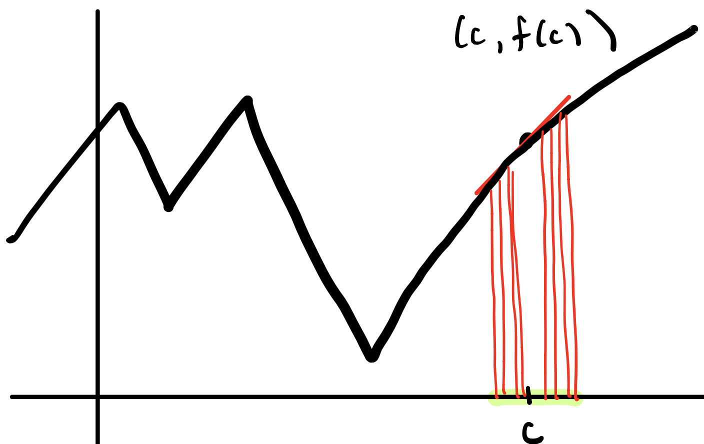
ex.



ONLY ONE
ABS MAX VALUE 1
(∞ MANY POINTS WHERE THIS VALUE IS ATTAINED)

ONLY ONE
ABS MIN VALUE -1
(∞ MANY POINTS WHERE THIS VALUE IS ATTAINED)

4 Fermat's Theorem If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.



IF $f(c)$ IS LOCAL MAX VALUE & $f'(c)$ EXISTS...

X IF $f'(c) > 0$

SLOPE OF TANGENT LINE > 0

f HAS SMALLER VALUES ON LEFT & LARGER VALUES ON THE RIGHT.

IMPOSSIBLE! $f(c)$ MAX VALUE.

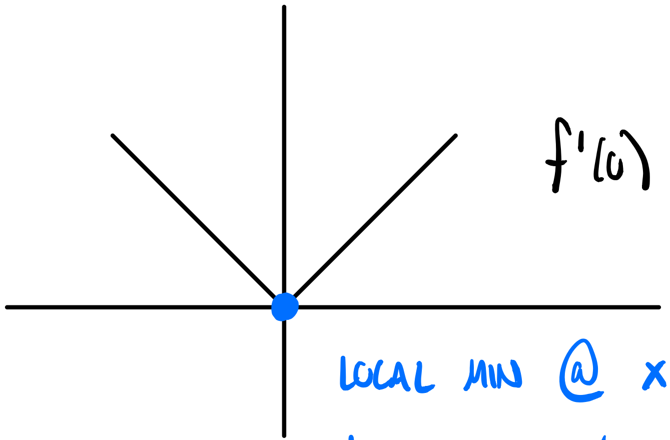
X SIMILARLY, $f'(c) < 0$ ... IMPOSSIBLE!

$\Rightarrow f'(c) = 0$ ✓

✓ $f(c)$ is a local extreme value & $f'(c)$ exists $\Rightarrow f'(c) = 0$

✗ $f(c)$ is a local extreme value $\not\Rightarrow f'(c) = 0$

e.g. $f(x) = |x|$



$f'(0)$ undefined.

local min @ $x = 0$

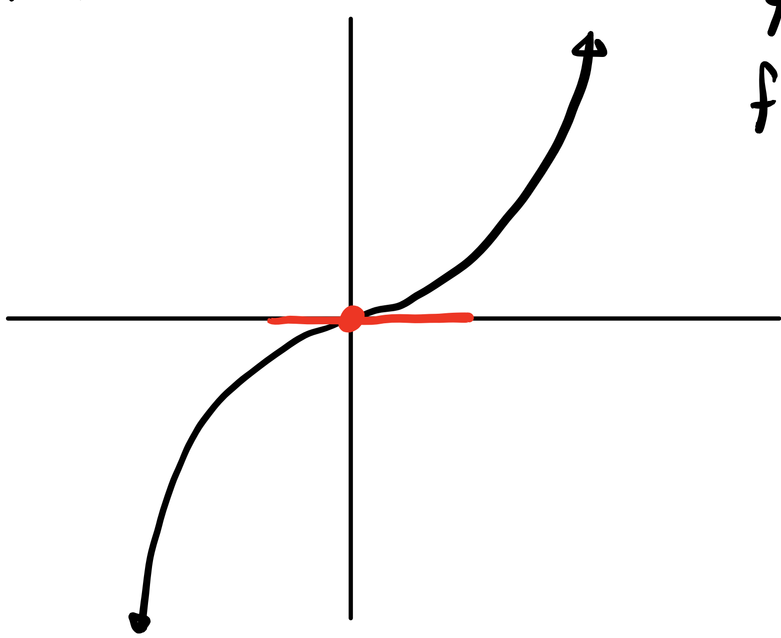
local min value $|0| = 0$

$f'(c) = 0 \not\Rightarrow f(c)$ is a local extreme value

e.g. $f(x) = x^3$

$f'(x) = 3x^2$

$f'(0) = 0$



FERMAT'S THEOREM: (ALT)

IF $f(c)$ IS A LOCAL MAX/MIN VALUE

THEN EITHER

- $f'(c)$ DOES NOT EXISTS, OR
- $f'(c) = 0$.

AKA "CRITICAL POINT"

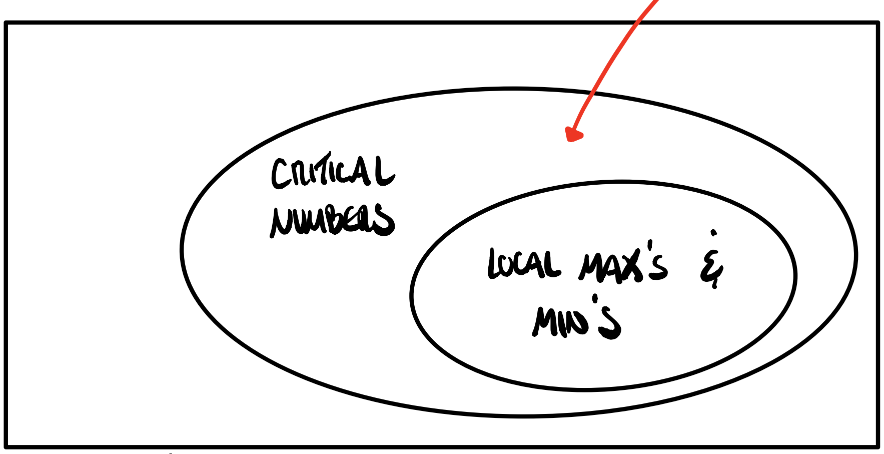
6 Definition A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

FERMAT'S THEOREM: (ALT)

IF $f(c)$ IS A LOCAL MAX/MIN VALUE

THEN c IS A CRITICAL POINT.

THERE ARE CRITICAL NUMBER THAT ARE NEITHER LOCAL MAX'S OR MIN'S.



DOM (f)

29–42 Find the critical numbers of the function.

29. $f(x) = 4 + \frac{1}{3}x - \frac{1}{2}x^2$

30. $f(x) = x^3 + 6x^2 - 15x$

31. $f(x) = 2x^3 - 3x^2 - 36x$

32. $f(x) = 2x^3 + x^2 + 2x$

33. $g(t) = t^4 + t^3 + t^2 + 1$

34. $g(t) = |3t - 4|$

39. $F(x) = x^{4/5}(x - 4)^2$

41. $f(\theta) = 2\cos\theta + \sin^2\theta$

39. $F'(x) = \frac{d}{dx}\left[x^{4/5}\right](x-4)^2 + x^{4/5}\frac{d}{dx}\left[(x-4)^2\right]$ PROD. RULE

$$= \frac{4}{5}x^{-1/5}(x-4)^2 + x^{4/5}\cdot 2(x-4)\cdot 1$$

$$= 2x^{-1/5}(x-4)\left[\frac{2}{5}(x-4) + x\right]$$

$$= \frac{2(x-4)\left(\frac{7}{5}x - \frac{8}{5}\right)}{x^{1/5}}$$

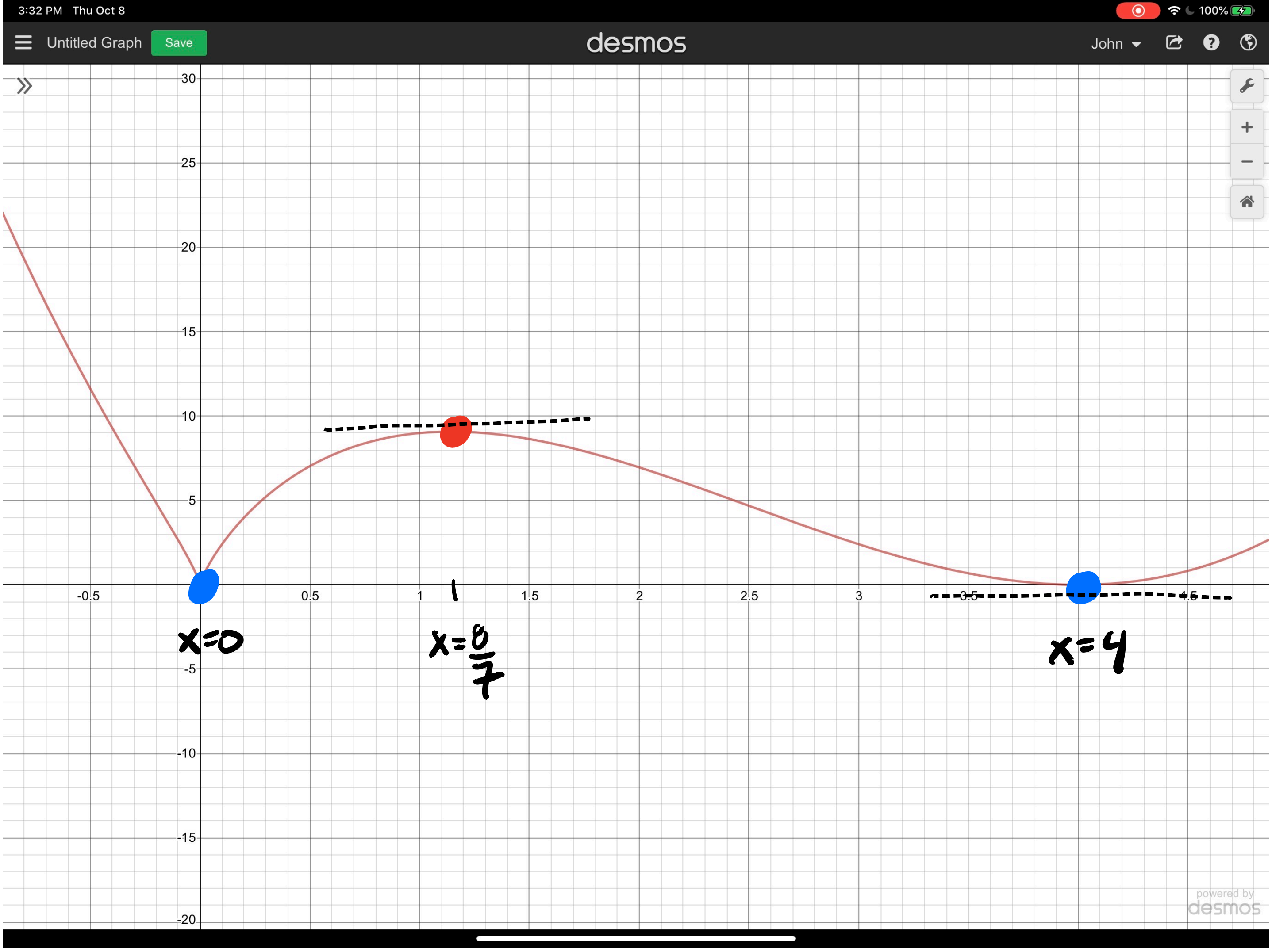
CRIT. NUM.

$$= \frac{2(x-4)(7x-8)}{5x^{1/5}} = \begin{cases} 0 \\ \text{UND.} \end{cases}$$

$F'(x) = 0$ WHEN $x = 4, \frac{8}{7}$

$F'(x)$ UND. WHEN $x = 0$ $(0 \in \text{DOM}(F))$

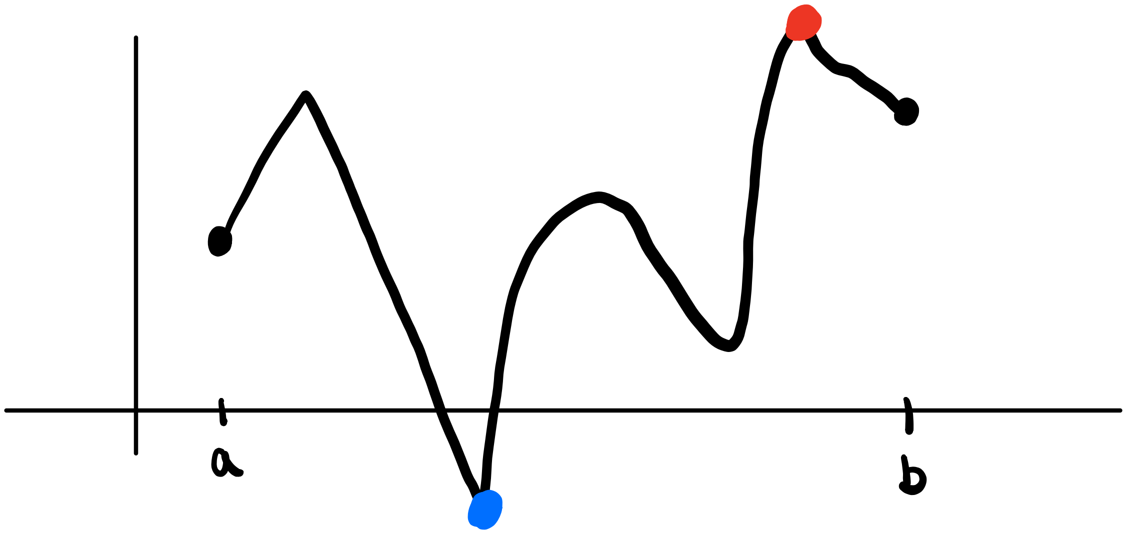
3 CRITICAL NUMBERS



3 **The Extreme Value Theorem** If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

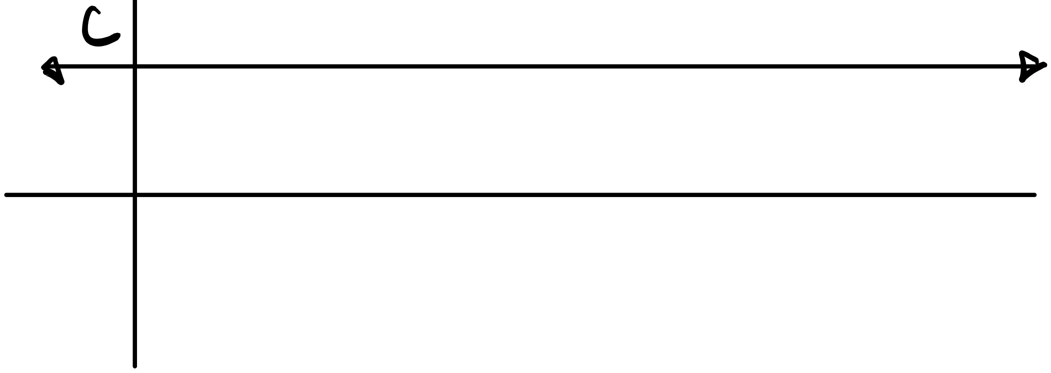
THINK ABOUT THE GRAPH:

CONTINUOUS f ON CLOSED INT. $[a, b]$

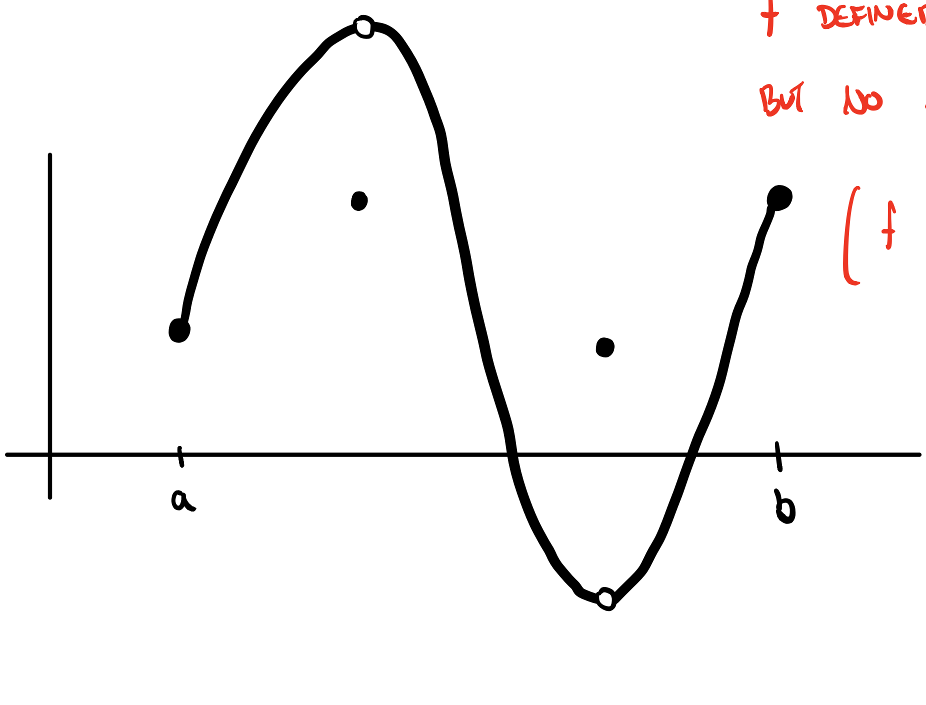


$f(x) = C$

C IS BOTH ABS MAX VALUE
& ABS MIN VALUE



ex. IF f IS <u>NOT</u> CONTINUOUS



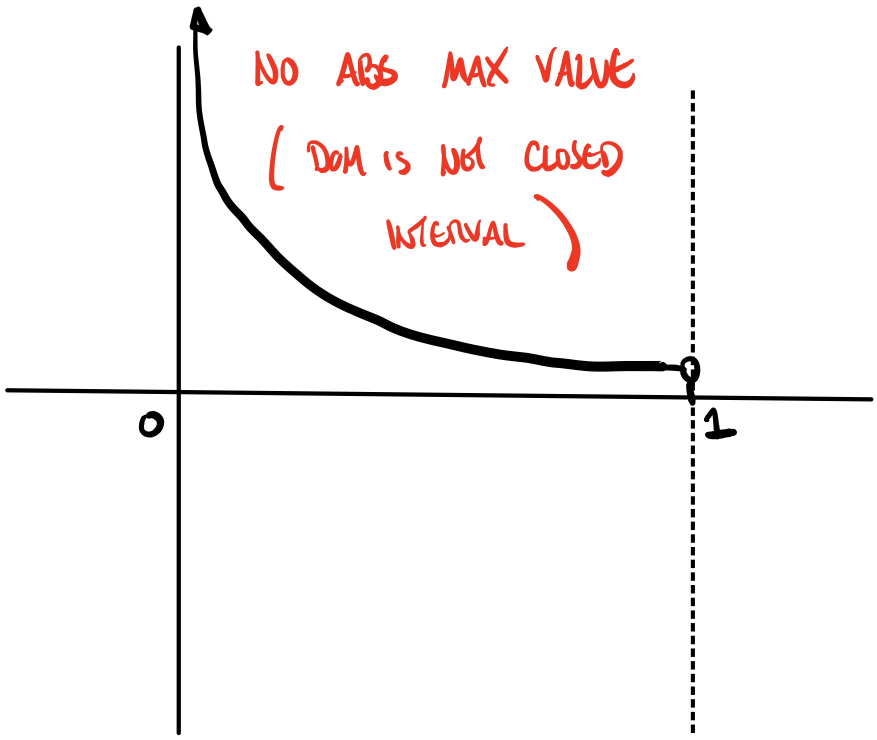
f DEFINED ON CLOSE INT $[a,b]$
BUT NO ABS MAX/MIN VALUE.

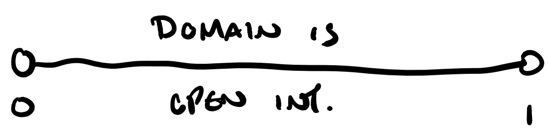
(f IS NOT CONTINUOUS)

ex. f is defined on an interval that is not closed, e.g. $(0, 1)$

$f(x) = \frac{1}{x}$

$0 < x < 1$





Next week:

Mon: No class Col. Day

Tue: Recitation

Wed: Lecture - Review for exam

Thur: Exam