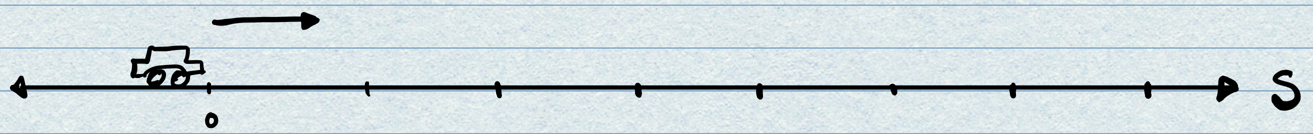
§1.4 THE TANGENT & VELOCITY PROBLEM



SUPPOSE A CAR DRIVES DOWN A STRAIGHT ROAD BEGINNING AT 12 PM. LET t = TIME PAST 12 PM, AND LET $s(t)$ = POSITION OF THE CAR t HOURS AFTER 12 PM.
(ROAD HAS DISTANCE MARKERS ON IT, FOR EXAMPLE)

t	$s(t)$ (IF FWD → ODOMETER)
0	0
1	45
2	110
3	180
3.5	222.5
3.9	244.2
3.99	249.41
4	250
4.01	250.59
4.1	255.8
4.5	277.5
5	320
6	400

$$\text{VELOCITY} = \frac{\text{DISTANCE}}{\text{TIME}}$$

e.g. mi/hr, ft/s, ETC.

$\underbrace{\text{AVERAGE VELOCITY FROM } t=0 \text{ TO } t=3}$

AVERAGE VELOCITY OVER FIRST 3 hrs OF THE TRIP.

$$\frac{\text{DISTANCE}}{\text{TIME}} = \frac{s(3)-s(0)}{3-0} = \frac{(180-0)\text{ mi}}{(3-0)\text{ hrs}}$$

$$= 60 \text{ mi/hr}$$

FIND AVERAGE VELOCITY FROM $t=3.5$ TO $t=4$

AVE VELOCITY $= \dfrac{\Delta s}{\Delta t} = \dfrac{s(4) - s(3.5)}{4 - 3.5}$

$$= \frac{250 - 222.5}{4 - 3.5} = \frac{27.5 \text{ mi}}{.5 \text{ hr}} = 55 \text{ mi/hr}$$

WHAT DOES THE SPEEDOMETER READ AT $t = 4$?

AVE VELOCITY FROM $t = 3.9$ TO $t = 4$

$$\frac{s(4) - s(3.9)}{4 - 3.9} = \frac{250 - 244.2}{4 - 3.9} = \frac{5.8 \text{ mi}}{.1 \text{ hr}}$$

$$= 58 \text{ mph}$$

AVE VELOCITY FROM $t = 3.99$ TO $t = 4$

$$\frac{s(4) - s(3.99)}{4 - 3.99} = \frac{250 - 249.41}{4 - 3.99} \frac{\text{mi}}{\text{hr}}$$

$$= \frac{.59}{.01} = 59 \text{ mph}$$

MORE ACCURATE APPROX:

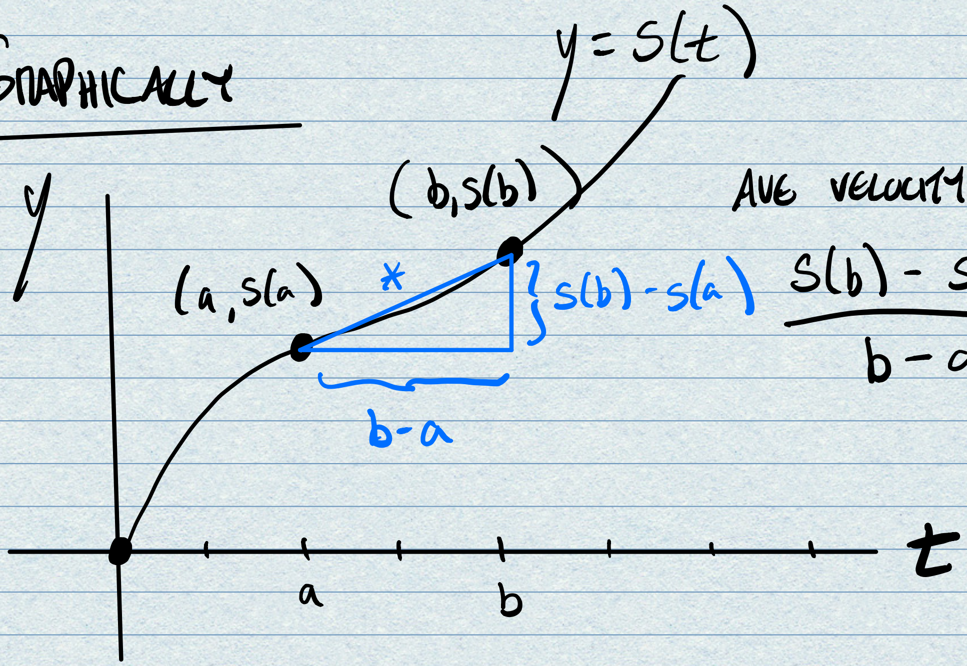
$$\frac{s(4) - s(4-h)}{4 - (4-h)}$$

WITH SMALLER & SMALLER VALUES OF h

$$\frac{s(4) - s(x)}{4 - x}$$ AS x GETS CLOSER AND CLOSER TO 4.

INSTANTANEOUS VELOCITY AT $t = 4$.

GRAPHICALLY

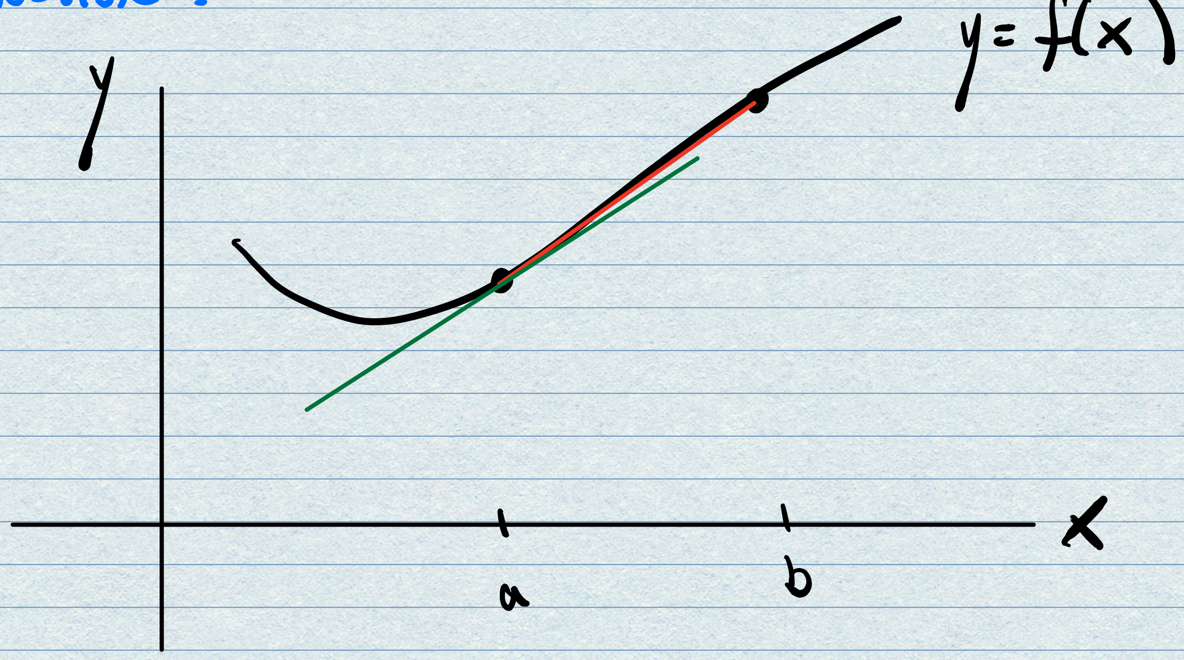


AVE VELOCITY OVER $a \le t \le b$

$$\frac{s(b) - s(a)}{b - a} = \frac{\Delta y}{\Delta x}$$

* SLOPE $\frac{\Delta y}{\Delta t} = \frac{s(b) - s(a)}{b - a}$ $\left(\frac{mi}{hr}\right)$

VELOCITY IS THE SLOPE OF THE POSITION FUNCTION.



DEF: AVERAGE RATE OF CHANGE OF f OVER $[a,b]$

IS $f_{AVE} = \dfrac{f(b)-f(a)}{b-a}$ *

(SLOPE OF LINE THROUGH $(a, f(a))$, $(b, f(b))$)

INSTANTANEOUS RATE OF CHANGE OF f AT a IS THE SLOPE OF THE TANGENT LINE TO $y = f(x)$ AT $(a, f(a))$. *

§ 1.5 THE LIMIT OF A FUNCTION

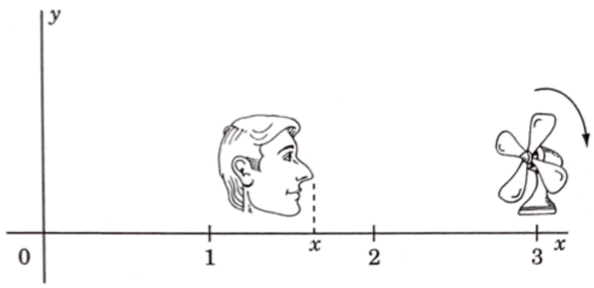


Figure 8.1 A dangerous limit to plug into.

We want to know what happens as x gets really close to 3—that is to say, what happens as your nose approaches 3, getting closer and closer, WITHOUT EVER ACTUALLY REACHING 3.

Well, what happens is that you feel a breeze that gets stronger as x gets closer to 3. We are interested in what happens to the amount of breeze as you approach 3. [We are taking $\lim_{x \to 3} b(x)$, where $b(x)$ is the breeze that you feel when your nose is at point x.]

Say you feel a breeze of 6 mph when $x = 2.9$, and the breeze increases as you move your nose toward the fan as in the following chart:

Nose position	2.9	2.99	2.999	2.9999	2.99999
Breeze	6	6.7	6.92	6.991	6.999993

It looks as if the breeze is approaching 7 mph as your nose approaches the fan. So we would say that

$$\lim_{x \to 3} b(x) = 7$$

You most certainly do not want to find out what happens when $x = 3$; you will feel a lot more than a breeze.

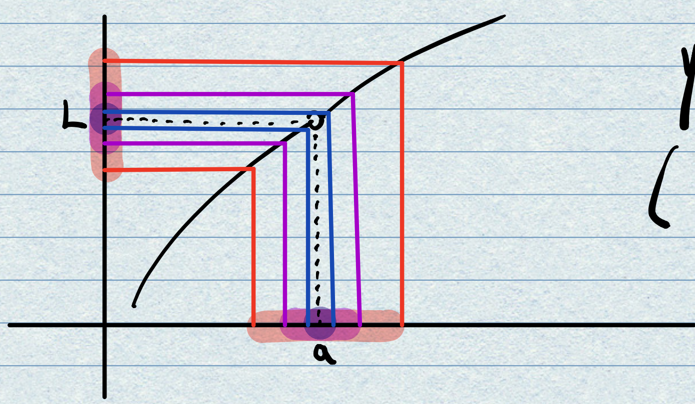
1 Intuitive Definition of a Limit Suppose $f(x)$ is defined when x is near the number a. (This means that f is defined on some open interval that contains a, except possibly at a itself.) Then we write

$$\lim_{x \to a} f(x) = L$$

and say "the limit of $f(x)$, as x approaches a, equals L"

if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by restricting x to be sufficiently close to a (on either side of a) but not equal to a.

THE OUTPUTS GET CLOSER & CLOSER TO L.



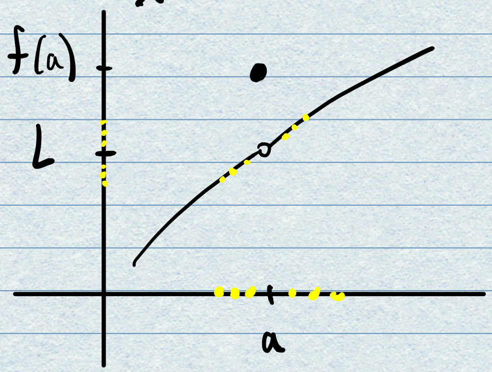
$y = f(x)$

$(f(a) = ?)$

AS THE INPUT X GETS CLOSER & CLOSER TO a...

WHAT HAPPENS WHEN $x = a$ HAS NO EFFECT ON

$$\lim_{x \to a} f(x).$$



$$\lim_{x \to a} f(x) = L$$

ex. LET $f(x) = \dfrac{x+2}{x^2-4}$

$$\lim_{x \to -2} f(x)$$

$(x \neq -2)$

x	$f(x)$
-1.9	
-1.99	
-1.999	
$\vdots$	
$\cancel{-2}$	? LIMIT
$\vdots$	
-2.001	
-2.01	
-2.1	

$$\lim_{x \to -2} \frac{x+2}{(x+2)(x-2)}$$

$(x \neq -2)$

NOTE: $\dfrac{\cancel{(x+2)}}{\cancel{(x+2)}(x-2)} = \dfrac{1}{x-2}$

FOR ALL VALUES OF x EXCEPT $x = -2$

LIMIT IS UNAFFECTED BY SUBSTITUTION...

$$\lim_{x \to -2} \frac{1}{x-2} = \boxed{-\frac{1}{4}}$$

x	$f(x)$
-2.001	$\frac{1}{-4.001} \approx -\frac{1}{4}$
WHEN x IS CLOSE TO -2	$f(x)$ IS CLOSE TO $-\frac{1}{4}$

MONDAY : NO CLASS

TUE : RECITATION

THUR : HW #2 DUE

(HW #3 DUE MON 9/14)