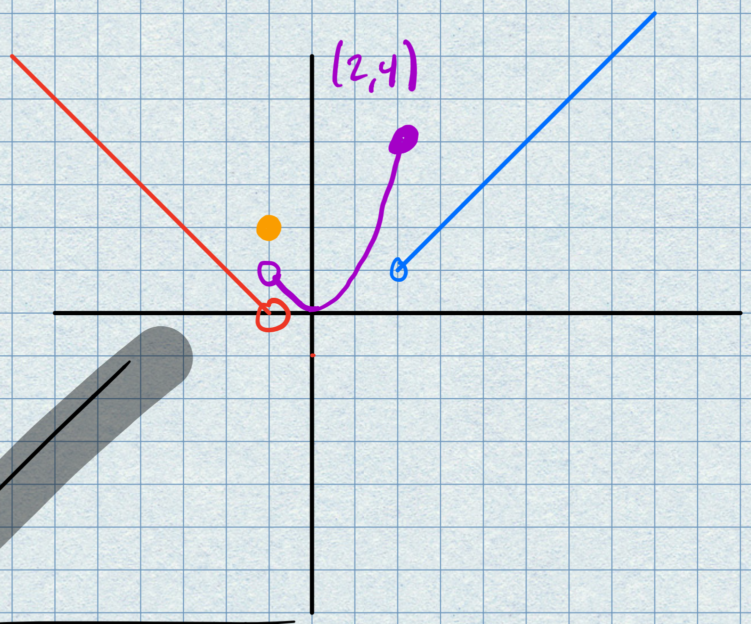
§ 1.1 CONTINUED

PIECEWISE DEFINED FUNCTIONS:

$$f(x) = \begin{cases} -x-1 & \text{IF } x < -1 \\ 2 & \text{IF } x = -1 \\ x^2 & \text{IF } -1 < x \le 2 \\ x-1 & \text{IF } x > 2 \end{cases}$$

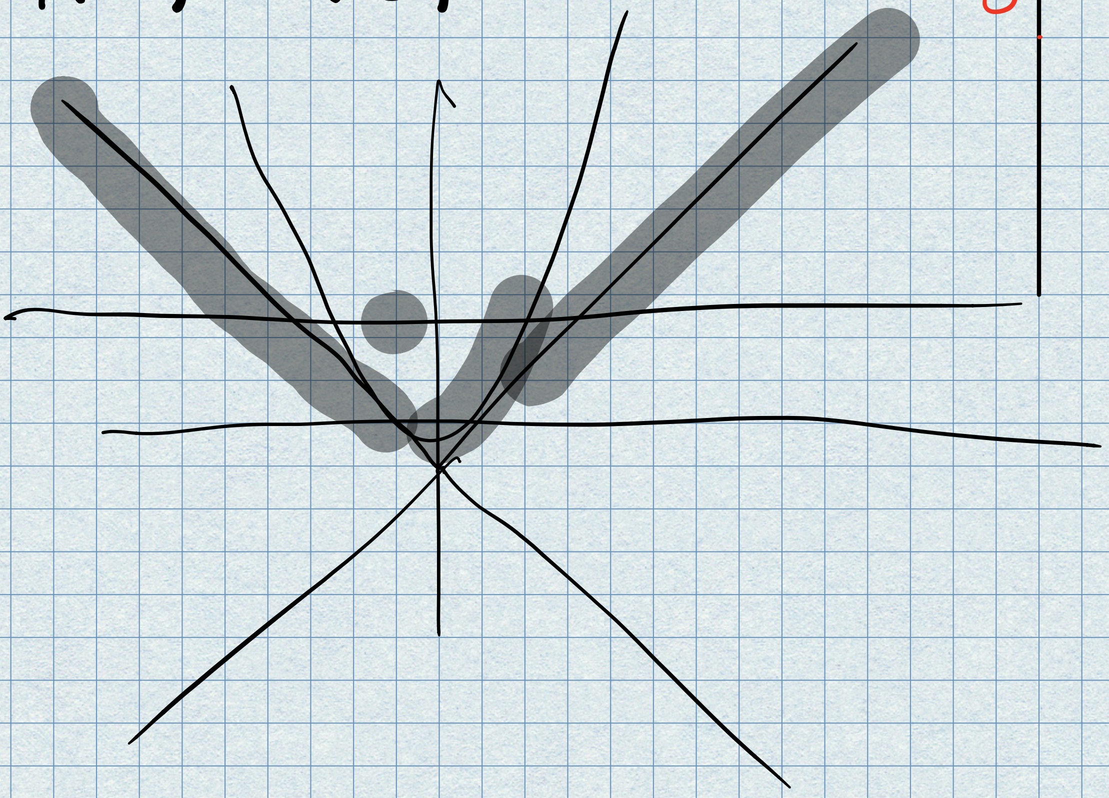
$y = -x - 1$

$y = 2$



$f(0) = 0^2 = 0$

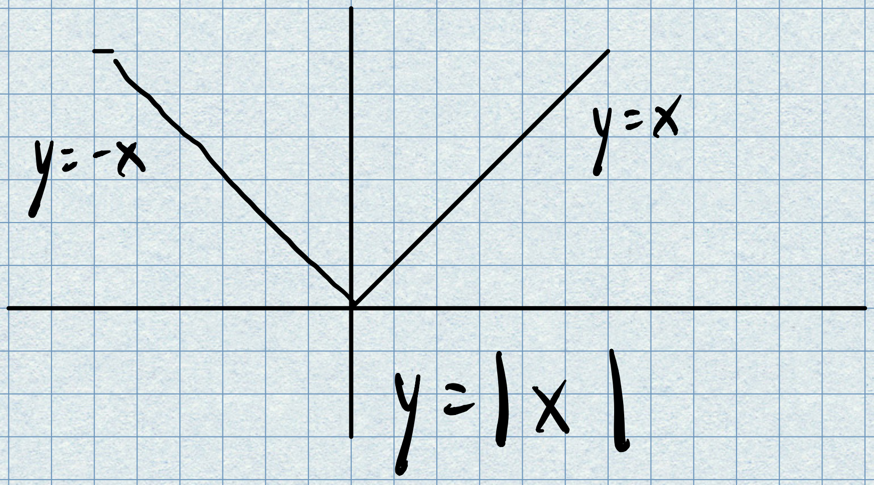
$f(-3) = -(-3) - 1 = 2$



Def: ABSOLUTE VALUE FUNCTION

$$f(x) = |x| = \begin{cases} x & \text{IF } x \ge 0 \\ -x & \text{IF } x < 0 \end{cases}$$

(NEVER NEGATIVE)



SYMMETRY

Def: A FUNCTION f IS <u>EVEN</u> IF $f(-x) = f(x)$ FOR ALL x IN THE DOMAIN OF f.

Def: f IS <u>ODD</u> IF $f(-x) = -f(x)$ FOR ALL x IN $\mathrm{Dom}(f)$.

ex. $f(x) = x^2$ IS EVEN:

$$f(-x) = (-x)^2 = (-1x)^2 = (-1)^2 x^2$$
$$= x^2 = f(x)$$

$g(x) = x^3$ IS ODD

$$g(-x) = (-x)^3 = (-1)^3 x^3 = -x^3$$
$$= -g(x)$$

POWER FUNCTION : $P(x) = x^n$, $n = 1, 2, 3, \ldots$

IF n IS EVEN $\Rightarrow P(x) = x^n$ IS EVEN

IF n IS ODD $\Rightarrow P(x) = x^n$ IS ODD

SUPPOSE f, g ARE EVEN FUNCTIONS

r, s ARE ODD FUNCTIONS.

CLASSIFY THE FOLLOWING COMBINATIONS AS EVEN OR ODD:

$f(x) + g(x)$ EVEN OR ODD?

$$f(-x) + g(-x) = f(x) + g(x) \quad \checkmark$$

ex. EVEN/ODD : $f(x) = x^4 - \dfrac{1}{x^2}$

·) SUM/DIFFERENCE OF EVEN FUNC. IS EVEN

·) SUM/DIFFERENCE OF ODD FUNC. IS ODD

ex. $f(x)r(x)$ EVEN OR ODD?

$$f(-x)r(-x) = f(x)(-r(x)) = -f(x)r(x)$$

ODD.

ex. $\dfrac{\text{EVEN}}{\text{ODD}}$: $\dfrac{f(x)}{r(x)}$ $\qquad \dfrac{x^4}{x^1} = x^3$

$$\frac{f(-x)}{r(-x)} = \frac{f(x)}{-r(x)} = -\frac{f(x)}{r(x)} \quad \text{ODD.}$$

ex. $f(x) = \dfrac{x^2+1}{x}$ ODD.

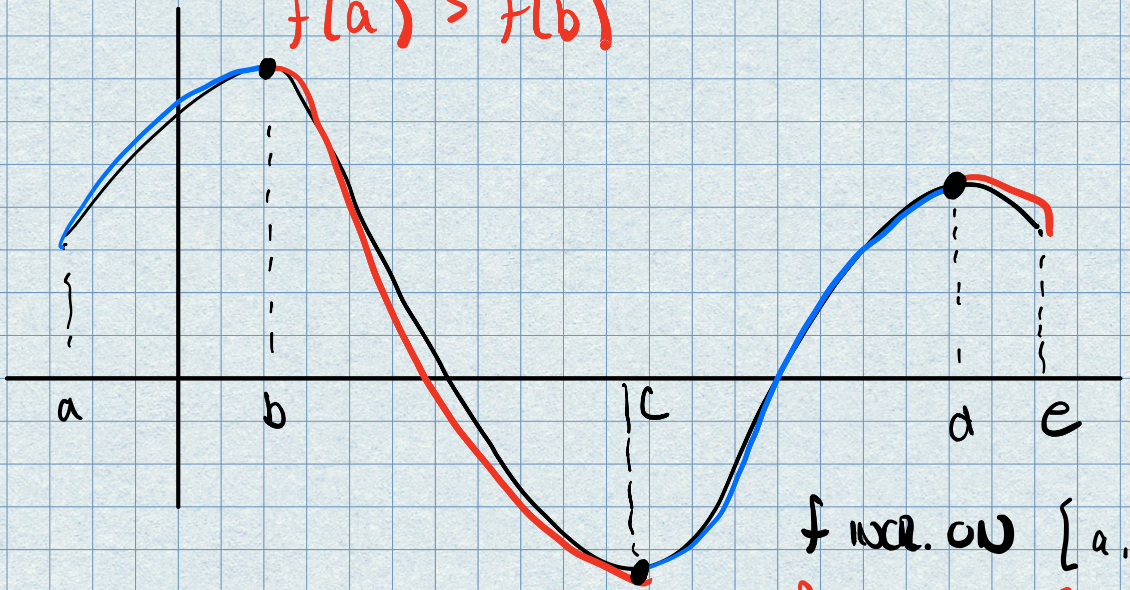
INCREASING / DECREASING

Def: FUNC f IS INCREASING / DECREASING ON AN INTERVAL I

IF FOR ANY $a < b$ IN I, WE HAVE

$f(a) < f(b)$

$f(a) > f(b)$



f INCR. ON $[a,b] \cup [c,d]$

f DECR ON $[b,c] \cup [d,e]$

§1.3 New Functions from Old Functions

Transformations of Functions

If we know the graph of $y=f(x)$
how is the graph of

$$y = a f(b(x+c)) + d$$

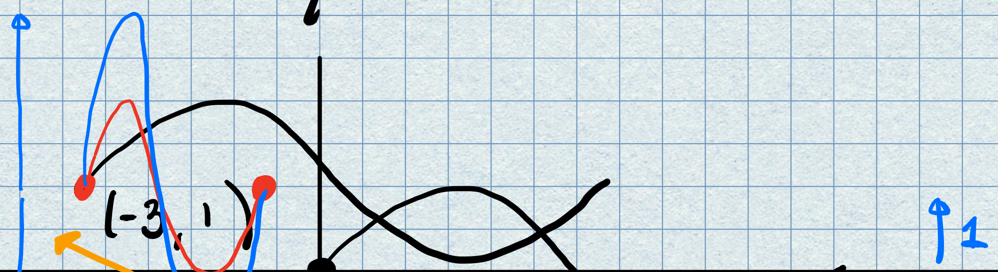
Take graph $y=f(x)$

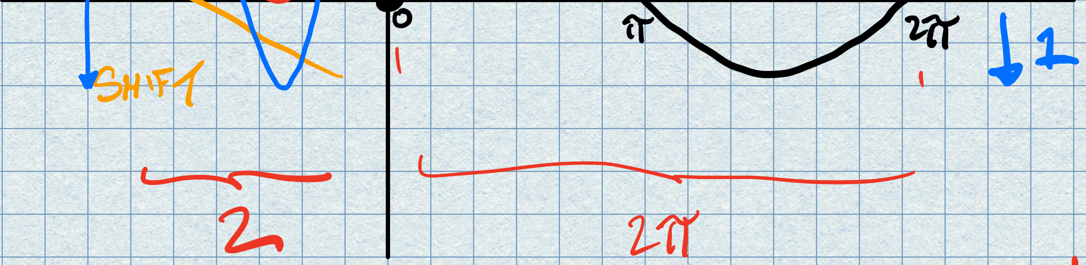
- shift horizontally by c
  (left if $c>0$, right if $c<0$)
- shift vertically by d
  (up if $d>0$, down in $d<0$)
- scale by a vertically
  (stretch / compress by a factor of a)
- scale horizontally by b
  (stretch / compress by a factor of b)

$|b|<1$ $\quad$ $|b|>1$ $\quad$ reflect if $b<0$

ex. $$\boxed{f(x) = 2\sin(\pi(x+3)) + 1}$$

Start with $y=\sin(x)$





HORIZONTAL DIST BETWEEN POINTS IS DIVIDED BY $b = \pi$

COMPOSITION OF FUNCTIONS:

$$f \circ g\,(x) = f(g(x))$$

$$x \longmapsto \boxed{g} \longmapsto g(x) \longmapsto \boxed{f} \longmapsto f(g(x))$$

ex. $f(x) = x + \frac{1}{x}$ $\qquad g(x) = \frac{x+1}{x-1}$

$$f \circ g\,(x) = f(g(x))$$

$$= \begin{cases} f\left(\frac{x+1}{x-1}\right) \\ g(x) + \frac{1}{g(x)} \end{cases} = \boxed{\frac{x+1}{x-1} + \frac{x-1}{x+1}}$$

$$\frac{x+1}{x-1} + \frac{1}{\frac{x+1}{x-1}}$$

$$f \circ f\,(x) = f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right) + \frac{1}{x + \frac{1}{x}}$$

$$f \circ f \circ f\,(x)$$

$$x \to \boxed{f} \to f(x) \to \boxed{f} \to f(f(x)) \to \boxed{f} \cdots$$

$\underbrace{f \circ f \circ \dots \circ f}_{n \text{ COMPOSITIONS}} (x)$

DYNAMICAL SYSTEMS:
IDEA IS TO LOOK FOR PATTERNS AS $n \to \infty$

ANOTHER WAY TO VISUALIZE FUNCTIONS

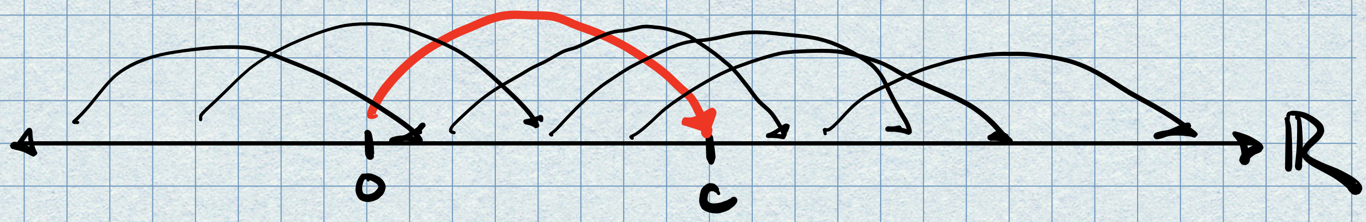
SAY $f : \mathbb{R} \to \mathbb{R}$ (REAL VALUED FUNC OF A REAL VARIABLE)

DOMAIN

CODOMAIN (TARGET)

ARROW DIAGRAM ON 1 NUMBER LINE

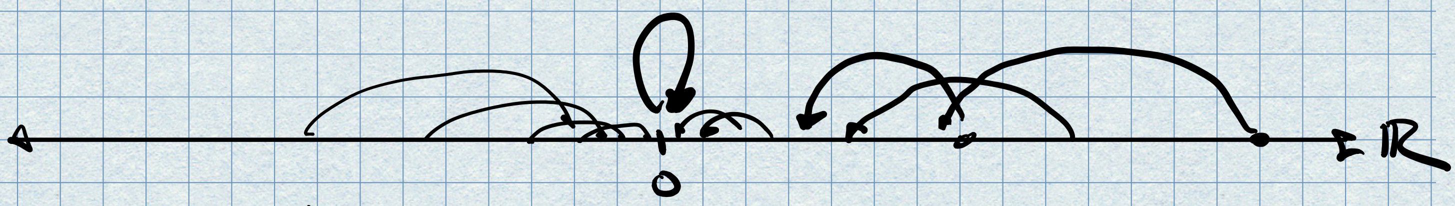
TRANSLATION



$f(x) = x + c, \quad c > 0$

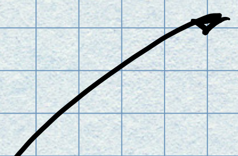
$f(x) = \frac{1}{2} x$

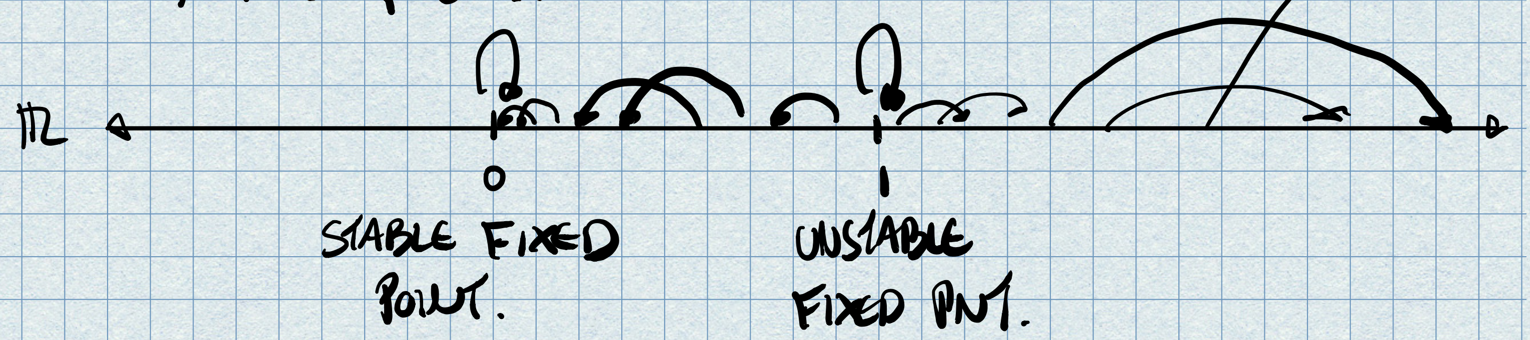
FIXED PNT: $f(x) = x$ (SOLN'S)



$f(f(f(0))) = 0$

ex. $f(x) = x^2, \ 0 \le x$





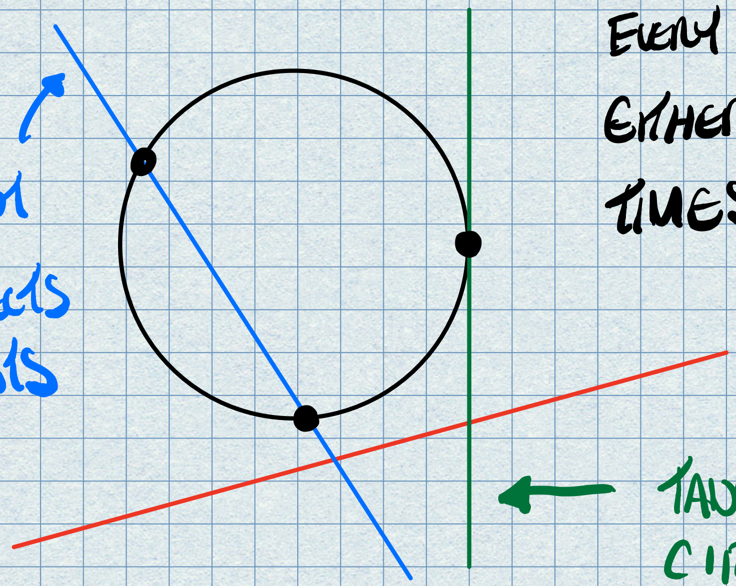
CHALLENG: CREATE ARROW DIAGRAM FOR

$$f(x) = \frac{1}{x}$$

$$g(x) = x + \sin x$$

§1.4 THE TANGENT & VELOCITY PROBLEM

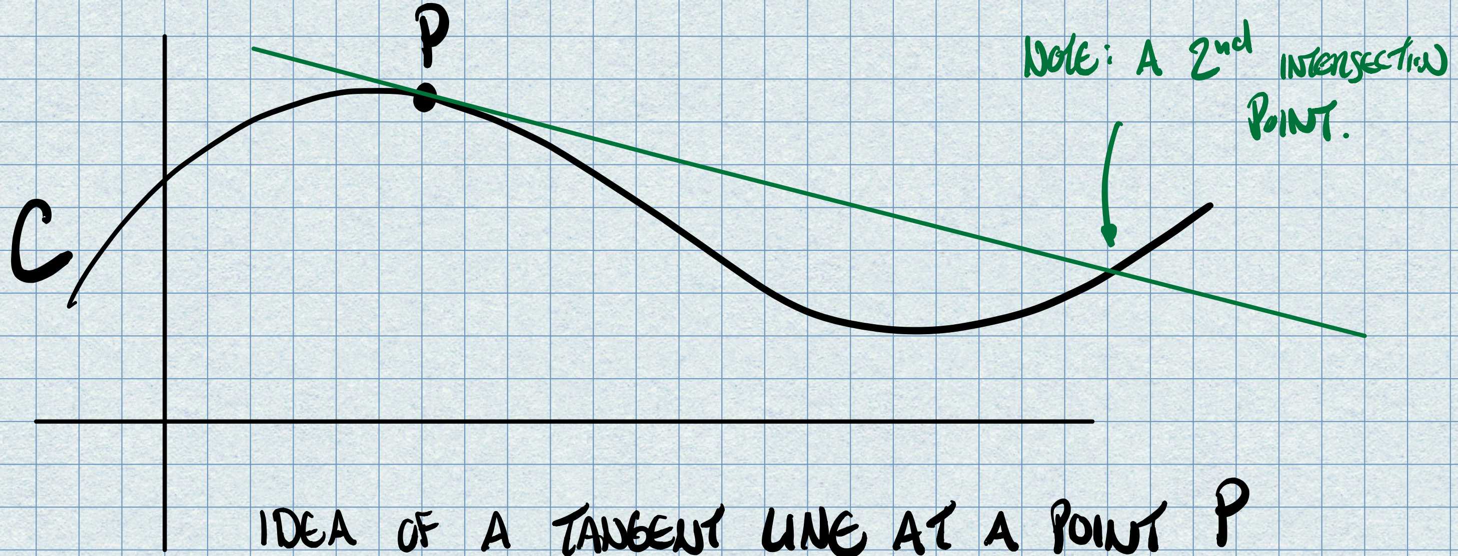
TANGENT LINE TO A CIRCLE



SECANT LINE CONNECTS 2 PNTS

EVERY LINE INTERSECTS A CIRCLE EITHER 0, 1, OR 2 TIMES

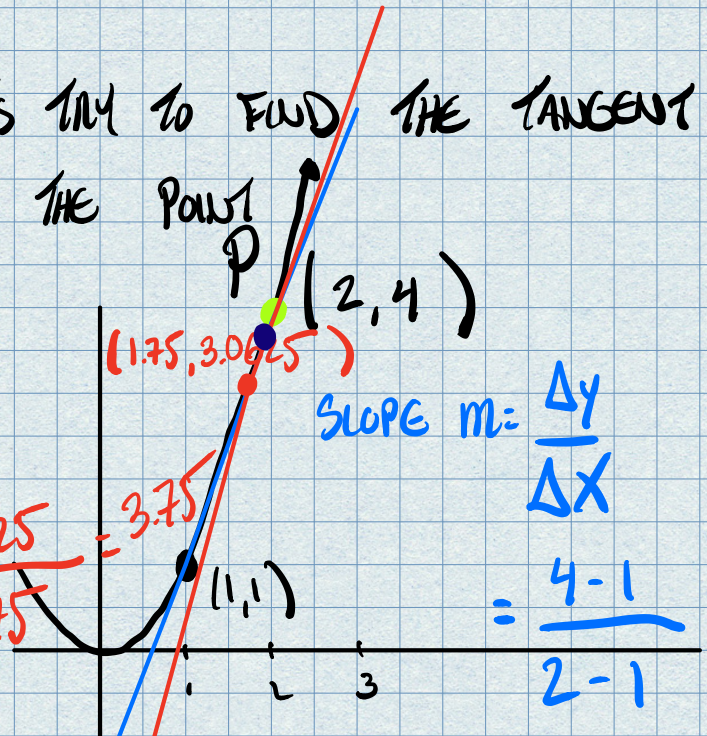
TANGENT LINE INTERSECTS CIRCLE EXACTLY ONCE



IDEA OF A TANGENT LINE AT A POINT P TO A CURVE C IS A STRAIGHT LINE THROUGH P WITH THE SAME DIRECTION AS C

WHEN WE ZOOM IN ON C AT P. THE CURVE STARTS TO STRAIGHTEN OUT & THE STRAIGHT LINE IT RESEMBLES AT A ZOOMED IN SCALE IS THE TANGENT LINE TO C AT P.

LET'S TRY TO FIND THE TANGENT LINE TO $y = x^2$ AT THE POINT



$m = \dfrac{\Delta y}{\Delta x} = \dfrac{4 - 3.0625}{2 - 1.75} = 3.75$

SLOPE $m = \dfrac{\Delta y}{\Delta x} = \dfrac{4-1}{2-1} = 3$

$y - 4 = m(x - 2)$

FIND m

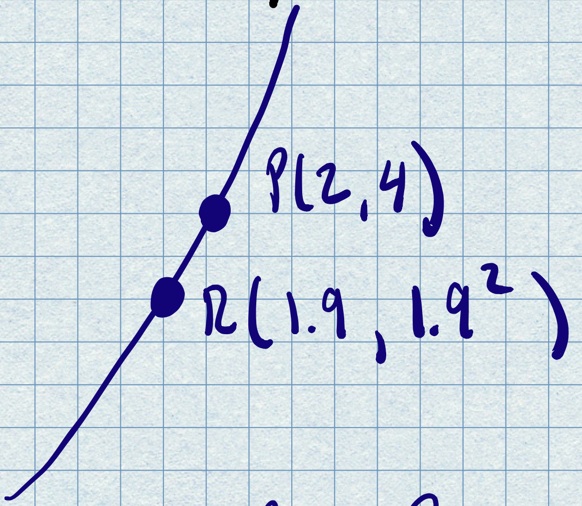
$m \approx 3$

$m \approx 3.75$

POINT-SLOPE FORMULA: m SLOPE

(a,b) POINT ON LINE

$$y-b=m(x-a)$$



SLOPE OF $\overline{PR}$

$$m=\frac{4-1.9^2}{2-1.9}=\frac{.39}{.1}=3.9$$

SLOPE $3 \rightarrow 3.75 \rightarrow 3.9$

AS WE MOVE R CLOSER & CLOSER TO P ALONG $y=x^2$, m GETS CLOSER & CLOSER TO 4. USING $m=4$

$\Longrightarrow$ TANGENT LINE $\boxed{y-4=4(x-2)}$