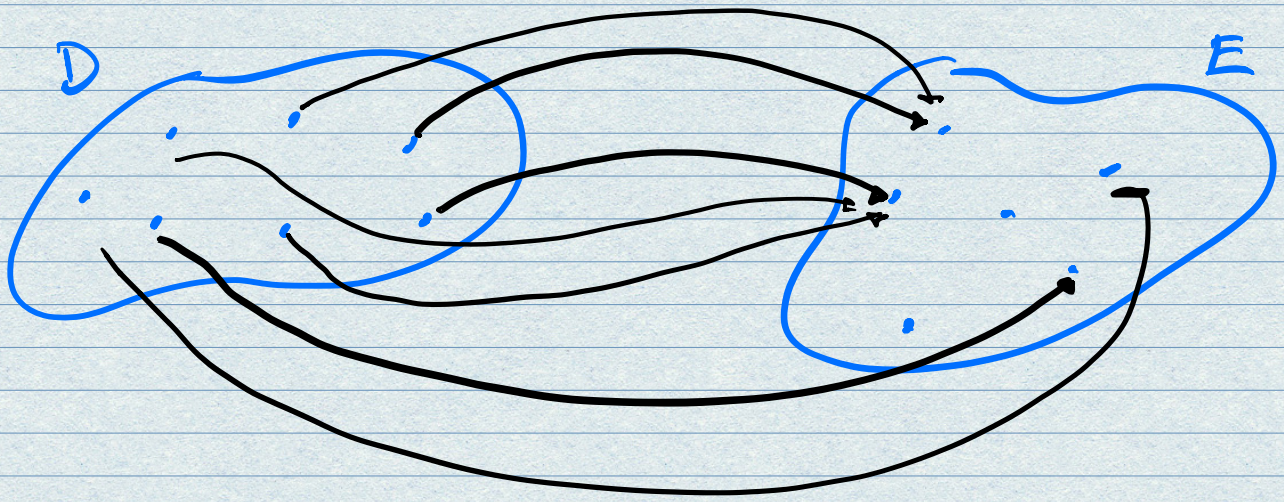
§1.1 FOUR WAYS TO REPRESENT A FUNCTION

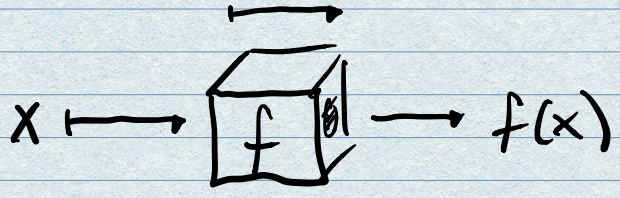
DARTBOARD

DEF: A FUNCTION f IS A RULE THAT ASSIGNS TO EACH ELEMENT x IN IN A SET D (DOMAIN) EXACTLY ONE ELEMENT $f(x)$ IN A SET E (CODOMAIN).



THE ELEMENTS IN CODOMAIN E THAT HAVE ELEMENTS IN THE DOMAIN ASSIGNED TO THEM (HIT BY FUNC. f) ARE CALLED THE **RANGE**

↑ POINT ON DARTBOARD HIT BY THE FUNCTION.



ex: $f(x) = x^2$ INPUT: x OUTPUT: $x^2 = (\text{INPUT})^2$

$f(2) = 2^2 = 4$ $\quad 2 \to f \to 4$

$f(a+1) = (a+1)^2 = (a+1)(a+1) = a^2 + 2a + 1$

ex. $g(x) = 4 + 3x - x^2$

"DIFFERENCE QUOTIENT"

EVALUATE & SIMPLIFY $\dfrac{g(3+h) - g(3)}{h}$

$$\frac{4 + 3(3+h) - (3+h)^2 - \left[4 + 3(3) - (3)^2\right]}{h}$$

$$\frac{4 + 9 + 3h - 9 - 6h - h^2 - 4 - 9 + 9}{h}$$

$$\frac{3h - 6h - h^2}{h} = \frac{h(3 - 6 - h)}{h}$$

$$= \boxed{-3 - h}$$

↑

FUNCTIONS f THAT ARE ALGEBRAICALLY DEFINED, BY DEFAULT, HAVE AS THEIR DOMAIN THE SET OF ALL REAL NUMBERS SUCH THAT $f(x)$ IS DEFINED.

ex. $f(x) = \sqrt{x}$. FIND $\text{DOM}(f)$

IN DOMAIN: $4, 9, 10$

NOT IN DOMAIN: $-1, -2, -\frac{1}{3}, \ldots$

$\text{Dom}(f) = \{ x \in \mathbb{R} \mid x \geq 0 \}$

SET NOTATION

"THE SET OF ALL x BELONGING TO THE SET OF REAL NUMBERS SUCH THAT $x \geq 0$."

ex. $g(x) = \dfrac{\sqrt{x}}{(x-2)(x-4)}$. FIND $\text{Dom}(g)$

NOTE: RADICANDS ≥ 0 (1)

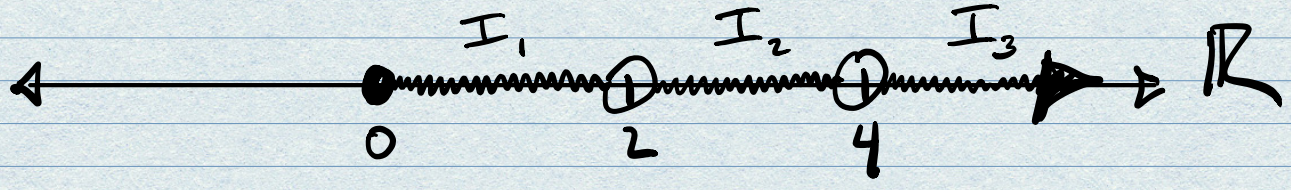
DENOMINATORS $\neq 0$

(1) $x \geq 0$

(2) $(x-2)(x-4) \neq 0$

$x-2 \neq 0$ $\quad$ $x-4 \neq 0$

$x \neq 2$ $\quad$ $x \neq 4$



$\text{Dom}(g) = \{ x \in \mathbb{R} \mid x \geq 0, x \neq 2, x \neq 4 \}$

$= [0, 2) \cup (2, 4) \cup (4, \infty)$

"UNION"

Def: THE GRAPH OF A FUNCTION f IS THE SET OF ALL ORDERED PAIRS (x, y) SUCH THAT THE EQUATION $y = f(x)$ IS SATISFIED.

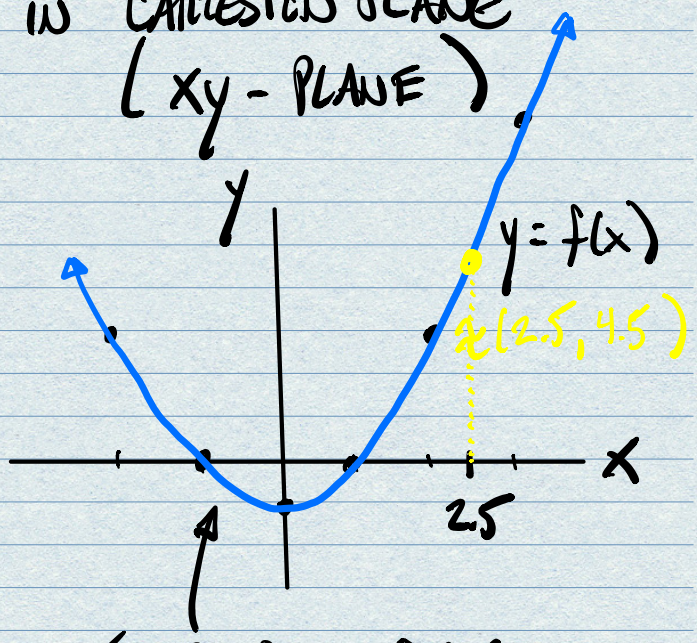
VISUALIZED AS POINTS IN CARTESIAN PLANE (xy-PLANE)

ex. $f(x) = x^2 - 1$

THE GRAPH OF f:

x	y
-2	3
-1	0
0	-1
1	0
2	3
3	8

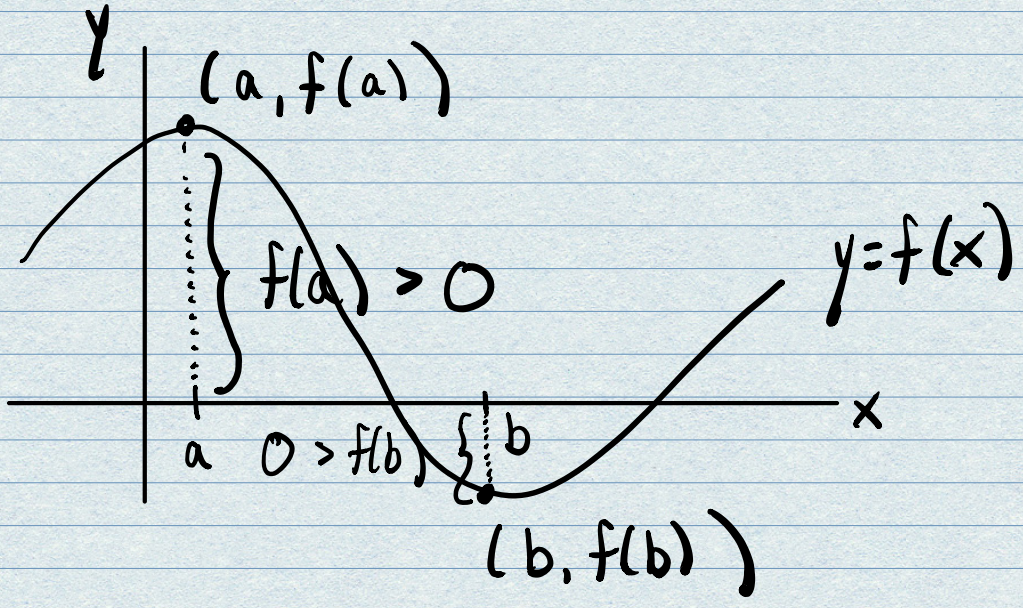
$y = x^2$



THIS PARABOLIC CURVE IS COMPOSED OF POINTS, EACH SATISFYING $y = x^2 - 1$

READING VALUES OF f FROM ITS GRAPH.

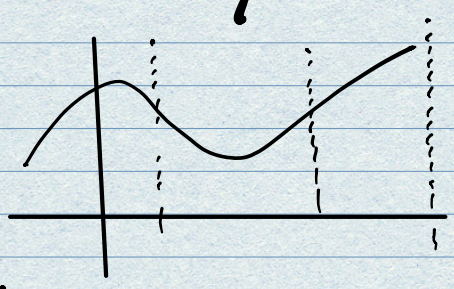
$f(2.5) \approx 4.5$ (5.25)



FOR MANY FUNCTIONS, THEIR GRAPHS ARE CURVES IN THE xy-PLANE.

IS EVERY CURVE IN xy-PLANE THE GRAPH OF A

FUNCTION?



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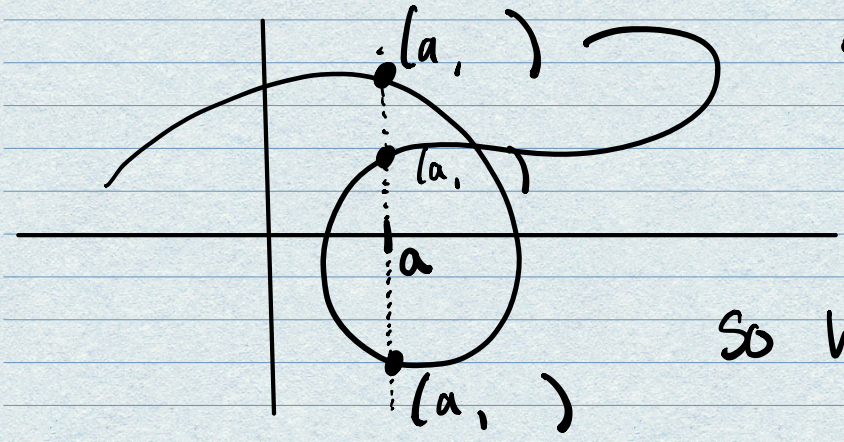

← THIS NOT THE GRAPH OF ANY FUNCTION

$y = f(x)$

SO WHAT IS $f(a)$

ONE INPUT a HAS 3 DIFFERENT OUTPUTS!!

$\Rightarrow$ NOT A FUNCTION.

VERTICAL LINE TEST

A CURVE IN XY-PLANE IS THE GRAPH OF A FUNCTION IF NO VERTICAL LINE INTERSECTS THE CURVE MORE THAN ONCE.