7.3 Basic Counting Priciples
Example/Discussion Problems

1. A sandwich shop lets customers create their own sandwich by repeatedly making choices from a selection of options as follows.

    (a) Choose a type of bread: white, wheat, rye, sourdough.

    (b) Choose a meat: tukey, beef, tuna.

    (c) Choose a cheese: Cheddar, none.

    How many different sandwiches can one order at this shop?

    > ### Multiplication Principle
    > Suppose n choices must be made, with
    > $$m_1 \text{ ways to make choice 1,}$$
    > $$m_2 \text{ ways to make choice 2,}$$
    > and so on, with
    > $$m_n \text{ ways to make choice } n.$$
    > Then there are
    > $$m_1 \cdot m_2 \cdot \cdots \cdot m_n$$
    > different ways to make the entire sequence of choices.

2. How many distinct 10 digit phone numbers exist? Note that neither the 1st nor 4th digit of a 10-digit phone number can be 0 or 1?

3. How many distinct 7 character license plates exist if the first 3 characters must be letters, and the last 4 characters must be digits?

4. How many distinct 7 character license plates exist if the first 3 characters must be letters, and the last 4 characters must be digits, and characters that appear next to each other must be different?

5. How many distinct 7 character license plates exist if the first 3 characters must be letters, and the last 4 characters must be digits, and no character can appear more than once?

6. A graphic designer has 256 colors from which to choose one color as a background, one color for largy type, and one color for small type. How many ways can this be done? (Do the choices need to be different?)

7. How many ways are there for a club with 15 members to choose one president, one vice-president, and one treasurer?

8. A mail deliverer needs to deliver packages to 11 different addresses. In how many different orders can they deliver the packages?

Factorial Notation

For any natural number n,

$$n! = n(n-1)(n-2)\cdots(3)(2)(1).$$

Also, by definition,

$$0! = 1.$$

Permutations

If $P(n,r)$ (where $r \leq n$) is the number of permutations of n elements taken r at a time, then

$$P(n,r) = \frac{n!}{(n-r)!}.$$

$P(n, n)$

The number of permutations of a set with n elements is $n!$; that is $P(n, n) = n!$.

9. How many ways are there to select 3 books from a collection of 8 books and arrange them on a shelf from left to right?

10. There are 8 democratic candidates running for mayor of NYC. On June 22nd, registered democrats will go to the polls and rank their top 5 choices for mayor. How many ways can registered democrats vote in the mayoral primary?

11. How many distinct ways can all of the letters in the followng words be arranged?

    (a) SWIMMING
    (b) BOOKKEEPER
    (c) MISSISSIPPI

    If the n objects in a permutation are not all distinguishable—that is, if there are n_1 of type 1, n_2 of type 2, and so on for r different types, then the number of **distinguishable permutations** is

$$\frac{n!}{n_1!\, n_2! \cdots n_r!}.$$

12. A *derangement* of n ordered objects is a rearrangemnt such that no object ends up in its original position. For example, the three-letter word CAT has two derangements: ATC and TCA. How many derangemnts does an n-letter word have when all n letters are distinct?