3.5.2 Systems of Linear Inequalities in Two Variables

- Solution Region
- We consider only systems with $\geq$ & $\leq$ (not $>$ or $<$)

Matched Problem 1 Solve the following system of linear inequalities graphically:

$$\begin{aligned} 3x + y &\leq 21 \\ x - 2y &\leq 0 \end{aligned}$$

DEFINITION Corner Point

A **corner point** of a solution region is a point in the solution region that is the intersection of two boundary lines.

Matched Problem 2 Solve the following system of linear inequalities graphically and find the corner points:

$$\begin{aligned} 5x + y &\geq 20 \\ x + y &\geq 12 \\ x + 3y &\geq 18 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

What if $\leq$ $\leq$ $\leq$?

Note: Not all intersections are corner points of the solution region.

(Unbounded) (Bounded)

DEFINITION Bounded and Unbounded Solution Regions

A solution region of a system of linear inequalities is **bounded** if it can be enclosed within a circle. If it cannot be enclosed within a circle, it is **unbounded**.

Solve the systems in Problems 29–38 graphically and indicate whether each solution region is bounded or unbounded. Find the coordinates of each corner point.

29. $\begin{aligned} 2x + 3y &\leq 12 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$

30. $\begin{aligned} 3x + 4y &\leq 24 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$

31. $\begin{aligned} 2x + y &\leq 10 \\ x + 2y &\leq 8 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$

32. $\begin{aligned} 6x + 3y &\leq 24 \\ 3x + 6y &\leq 30 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$

33. $\begin{aligned} 2x + y &\geq 10 \\ x + 2y &\geq 8 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$

34. $\begin{aligned} 4x + 3y &\geq 24 \\ 3x + 4y &\geq 8 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$

35. $\begin{aligned} 2x + y &\leq 10 \\ x + y &\leq 7 \\ x + 2y &\leq 12 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$

36. $\begin{aligned} 3x + y &\leq 21 \\ x + y &\leq 9 \\ x + 3y &\leq 21 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$

Applications

51. Water skis. A manufacturing company makes two types of water skis, a trick ski and a slalom ski. The trick ski requires 6 labor-hours for fabricating and 1 labor-hour for finishing. The slalom ski requires 4 labor-hours for fabricating and 1 labor-hour for finishing. The maximum labor-hours available per day for fabricating and finishing are 108 and 24, respectively. If x is the number of trick skis and y is the number of slalom skis produced per day, write a system of linear inequalities that indicates appropriate restraints on x and y. Find the set of feasible solutions graphically for the number of each type of ski that can be produced.

52. Furniture. A furniture manufacturing company manufactures dining-room tables and chairs. A table requires 8 labor-hours for assembling and 2 labor-hours for finishing. A chair requires 2 labor-hours for assembling and 1 labor-hour for finishing. The maximum labor-hours available per day for assembly and finishing are 400 and 120, respectively. If x is the number of tables and y is the number of chairs produced per day, write a system of linear inequalities that indicates appropriate restraints on x and y. Find the set of feasible solutions graphically for the number of tables and chairs that can be produced.

Given 3 farmers, where do they build fences? 4? 5?