1 Bernoulli Trials and Binomial Experiments

DEFINITION Bernoulli Trials

A sequence of experiments is called a **sequence of Bernoulli trials**, or a **binomial experiment**, if

1. Only two outcomes are possible in each trial.
2. The probability of success p for each trial is a constant (probability of failure is then $q = 1 - p$).
3. All trials are independent.

↳ $P(A|B) = P(A)$

↳ RESULTS OF PREVIOUS TRIALS

Definition A **binomial experiment** is one that has these five characteristics:

1. The experiment consists of n identical trials. (BERNOULLI TRIALS)
2. Each trial results in one of two outcomes. For lack of a better name, the one outcome is called a success, S, and the other a failure, F.
3. The probability of success on a single trial is equal to p and remains the same from trial to trial. The probability of failure is equal to $(1 - p) = q$.
4. The trials are independent.
5. We are interested in x, the number of successes observed during the n trials, for $x = 0, 1, 2, \ldots, n$.

1. Label each of the following experiments as binomial or not binomial.
   (a) A single coin is flipped repeatedly until a head is observed and x is the number of flips.
   (b) Seven cards are dealt from a shuffled deck of 52 cards and x is the number of aces dealt.
   (c) Due to a pandemic, only 1 out of every 5 customers is allowed into a particular store. Sarah visit this store on 7 consecutive days and x is the number times she is allowed into the store.
   (d) A jar contains 20 marbles: 12 red and 8 blue. Jessica selects 5 marbles from the jar simulteously and x is the number of red marbles.
   (e) A jar contains 20 marbles: 12 red and 8 blue. Jessica selects 5 marbles from the jar, replacing the marble after each selection, and x is the number of red marbles.

(a) FLIPPING A COIN IS A BERNOULLI TRIAL

(TWO POSSIBLE OUTCOMES)

PROB. OF SUCCESS/FAILURE ON EACH TRIAL IS INDEPENDENT OF ALL OTHER TRIALS.

BUT $x \neq$ # OF SUCCESSES IN n TRIALS

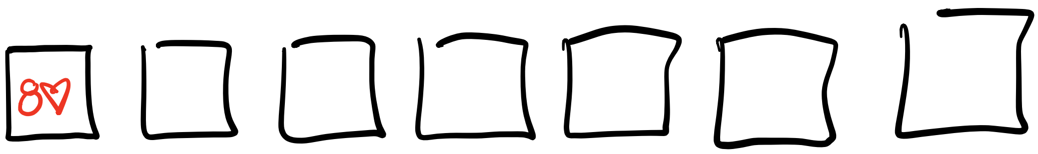
SUPPOSED TO BE PRE-DETERMINED!

THIS GIVES FINITE RANGE FOR POSSIBLE VALUES OF x (# SUCESSES)

0 MIN

n MAX

(b) Seven cards are dealt from a shuffled deck of 52 cards and x is the number of aces dealt.



ACE $\frac{4}{52}$ ACE $\frac{4}{51}$

NOT ACE $\frac{48}{52}$ NOT ACE $\frac{47}{51}$

THE TRIALS ARE NOT INDEPENDENT.

(c) Due to a pandemic, only 1 out of every 5 customers is allowed into a particular store. Sarah visit this store on 7 consecutive days and x is the number times she is allowed into the store.

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(c) YES, (ASSUMING INDEPENDENT TRIALS, REASONABLE)

(d) A jar contains 20 marbles: 12 red and 8 blue. Jessica selects 5 marbles from the jar simulteously and x is the number of red marbles. TRIALS NOT INDEPENDENT

(e) A jar contains 20 marbles: 12 red and 8 blue. Jessica selects 5 marbles from the jar, replacing the marble after each selection, and x is the number of red marbles. ✓

TRIALS IDENTICAL.

successes in n trials → $P(\text{success}) = p$, $P(\text{failure}) = q$

DEFINITION Binomial Distribution

Exponents add up to n

$$P(X_n = x) = P(x \text{ successes in } n \text{ trials})$$
$$= {}_nC_x p^x q^{n-x} \qquad x \in \{0, 1, 2, \ldots, n\}$$

where p is the probability of success and q is the probability of failure on each trial. Informally, we will write $P(x)$ in place of $P(X_n = x)$.

failures.

2. Imagine two different six-sided fair dice, called die A and die B.

   - Die A has its faces labeled 1, 1, 1, 2, 2, 3.
   - Die B has its faces labeled 1, 2, 2, 3, 3, 3.

   Which of the following events is more likely? Why?

   (a) Roll die A 5 times and roll a 2 exactly 3 times.

   (b) Roll die B 12 times and roll a 3 exactly 7 times.

   (c) Roll both dice simulataneously 9 times and roll doubles exactly 6 times.

(a) roll 2 : success → $p = P(\text{success}) = \frac{1}{3}$

$q = P(\text{failure}) = 1 - p = \frac{2}{3}$

$n = \#\text{ trials} = 5$

$$P(x=3) = {}_5C_2 \left(\tfrac{1}{3}\right)\left(\tfrac{1}{3}\right)\left(\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right)\left(\tfrac{2}{3}\right)$$

ways to get 2 successes in 5 trials

___ p ___ ___ p

$$P(x=3) = {}_5C_2 \left(\tfrac{1}{3}\right)^2 \left(\tfrac{2}{3}\right)^3 = .3292$$

2. Imagine two different six-sided fair dice, called die A and die B.

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COUNTING SUCCESSES : ROLL A 3

(b) $n = 12$

$p = .5$

$q = .5$

$$P(x=7) = {}_{12}C_7 \, (.5)^7 (.5)^5 = {}_{12}C_7 \, (.5)^{12}$$

$$\approx .1934$$

$$P(x=k) = {}_nC_k \, p^k q^{n-k}$$

(c) SUCCESS = ROLL DOUBLES

$$\underset{A\ B}{(1,1)} \cup \underset{A\ B}{(2,2)} \cup \underset{A\ B}{(3,3)}$$

$$\left(\tfrac{1}{2}\right)\left(\tfrac{1}{6}\right) + \left(\tfrac{1}{3}\right)\left(\tfrac{1}{3}\right) + \left(\tfrac{1}{6}\right)\left(\tfrac{1}{2}\right) = \frac{15}{54} = \frac{5}{18}$$

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COUNTING SUCCESS $\Rightarrow$

$p = \frac{5}{18}$

$q = 1 - \frac{5}{18} = \frac{13}{18}$

$n = 9$

$$P(x=6) = {}_9C_6 \left(\tfrac{5}{18}\right)^6 \left(\tfrac{13}{18}\right)^3$$

$$\approx .0145$$

When x is the number of successes in a series of n Bernoulli trials, the mean and standard deviation for x are as follows.

Mean: $\mu = np$

Standard deviation: $\sigma = \sqrt{npq}$

FLIP COIN $n=10$ TIMES, $x=$ # HEADS.

$\mu = (10)(.5) = 5$ HEADS

3. Let x represent be the number of success in 20 Bernoulli trials, each with probability of success $p = .85$. Find the mean (i.e. expected value) and standard deviation for x.

MEAN $\mu = (20)(.85) = 17$

STANDARD DEVIATION $\sigma = \sqrt{(20)(.85)(.15)}$

$= \sqrt{2.55}$

$= 1.5969$

2 Normal Distributions

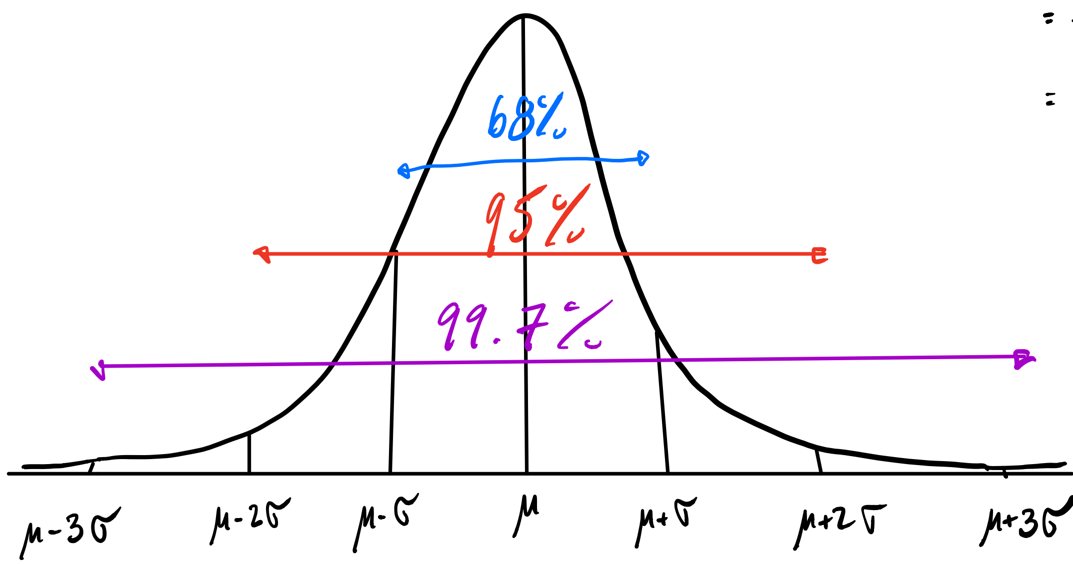


Figure 1: The 68-95-99.7 rule for normal distributions.

$= \sqrt{2}$

$= 1$

4. A machine in a bottling plant is set to dispense 12 oz of soda into cans. The machine is not perfect, and so every time the machine dispenses soda, the exact amount dispensed is a number x with a normal distriution. The mean and standard deviation for x are $\mu = 12$ oz and $\sigma = 0.15$ oz, respectively. Approximate the following probabilities using the 68-95-99.7 rule.

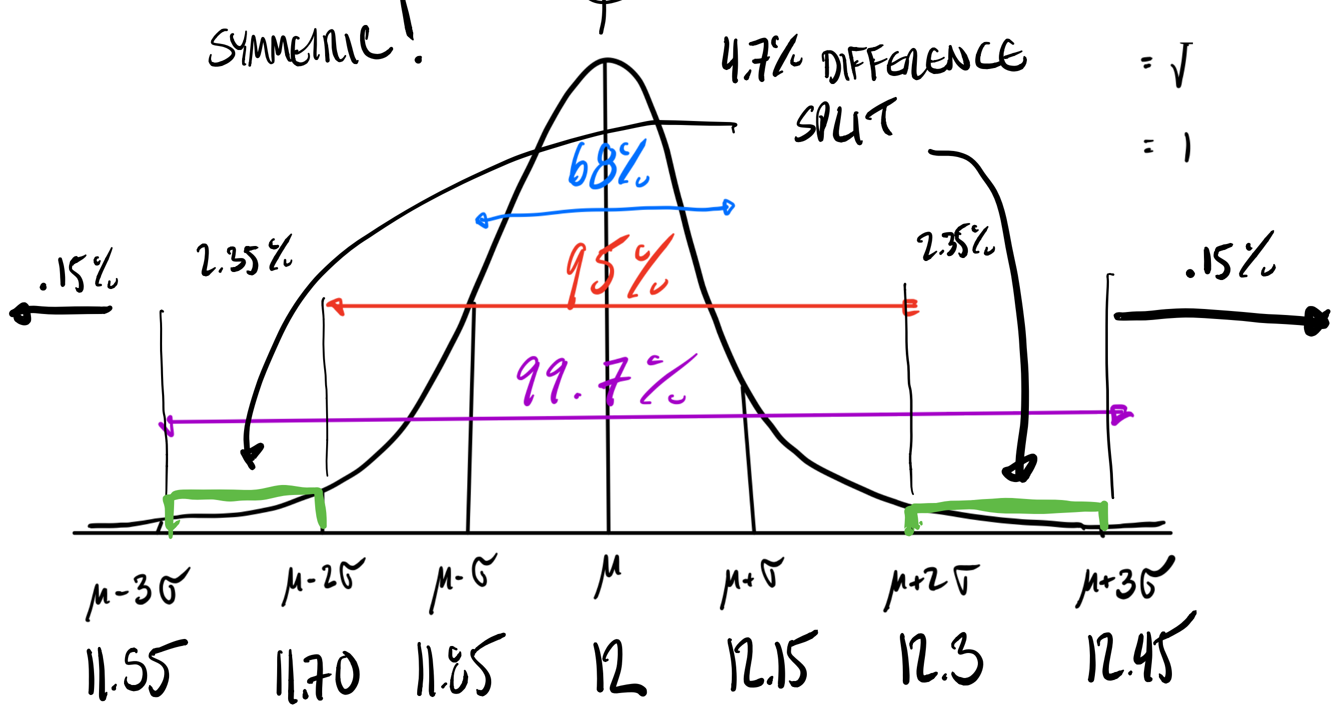
   (a) $P(11.85 \leq x \leq 12.15)$

   (b) $P(11.70 \leq x \leq 12)$

   (c) $P(x \leq 11.70)$

   (d) $P(12.3 \leq x \leq 12.45)$

   (e) $P(x \leq 12 \cup x \geq 12.45)$



(a) $P(11.85 \leq x \leq 12.15)$ $= .68$ or 68%

(b) $P(11.70 \leq x \leq 12)$ $= \frac{1}{2} P(11.70 \leq x \leq 12.30) = \frac{1}{2}(.95) = .475$ or 47.5%

(c) $P(x \leq 11.70)$ $= \frac{1}{2}(1 - P(11.7 \leq x \leq 12.3)) = \frac{1}{2}(1 - .95) = .025$ or 2.5%

(d) $P(12.3 \leq x \leq 12.45)$ $= .0235$ or 2.35%

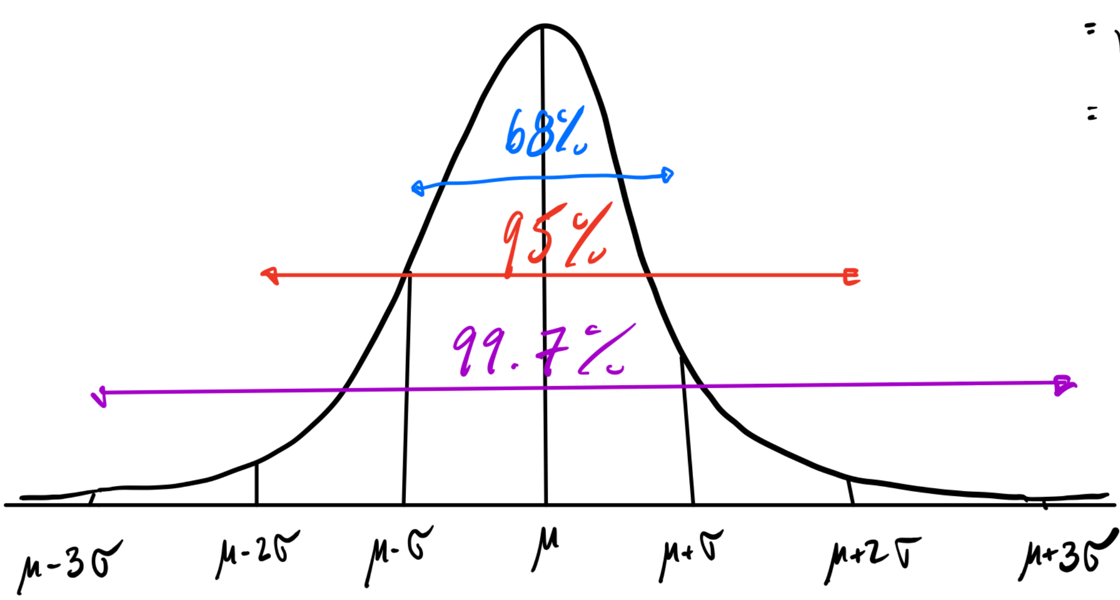
(e) $P(x \leq 12 \cup x \geq 12.45)$ $= P(x \leq 12) + P(x \geq 12.45)$

$.5 + .0015 = .5015$

50% $.15\%$ $= 50.15\%$

$= \sqrt{}$

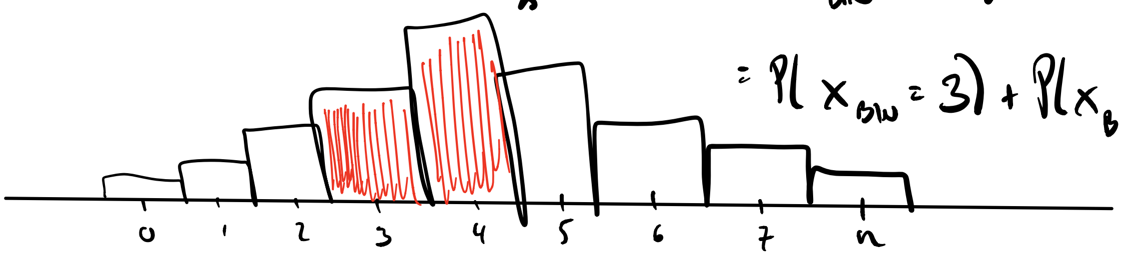
$= 1$



NOTE: PROBABILITY DISTRIBUTION FOR BINOMIAL EXPERIMENT IS DISCRETE $(x = 0, 1, 2, \ldots, n)$

$P(3 \leq X_{BIN} \leq 4)$

$= P(X_{BIN} = 3) + P(X_{BIN} = 4)$



NORMAL PROBABILITY DISTRIBUTION IS CONTINUOUS

MEASUREMENTS X THAT CAN TAKE ANY VALUE.

(INCLUDING DECIMALS, FRACTIONS, ETC.)

$P(3 \leq X_{NORM} \leq 4)$

AREA UNDER CURVE BETWEEN X=3, X=4.