Finite Math, MATH 1100

Exercises review 2 (for the second midterm)

Solutions

1. (a) $26^4 = \boxed{456,976}$.
   (b) $26 \cdot 25 \cdot 24 \cdot 23 = \boxed{358,800}$.
2. (a) $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{120}$.
   (b) $4 \cdot 3 \cdot 2 \cdot 1 = \boxed{24}$.
3. (a) $C(12,4) = \boxed{495}$.
   (b) $C(9,4) = \boxed{126}$.
   (c) $C(9,4) + C(9,3) \cdot C(3,1) = 126 + 84 \cdot 3 = \boxed{378}$.
   (d) $P(12,4) = 12 \cdot 11 \cdot 10 \cdot 9 = \boxed{11,880}$.
4. (a) Using combinations: $\dfrac{C(26,2)}{C(52,2)} = \dfrac{325}{1,326} = \boxed{0.245}$. (Alternatively, using the general multiplication rule: $\dfrac{26}{52} \cdot \dfrac{25}{51} = 0.245$.)
   (b) $\dfrac{C(12,1) \cdot C(40,1)}{C(52,2)} = \dfrac{12 \cdot 40}{1,326} = \boxed{0.36}$.
5. $\dfrac{C(2,1) \cdot C(11,2)}{C(13,3)} = \dfrac{2 \cdot 55}{286} = \boxed{0.385}$.
6. (a) There are 12 face cards in a standard deck. We have
   $$p = P(\text{at least one face card}) = 1 - P(\text{no face cards}) = 1 - \frac{40}{52} \cdot \frac{39}{51} \cdot \frac{38}{50} = \boxed{0.553}.$$
   (b) Let X be the number of times you win. Then X is a binomial random variable with $N = 10$ and $p = 0.553$. Therefore
   $$P(X = 6) = C(10,6)(0.553)^6 \cdot (1 - 0.553)^4 = 210 \cdot (0.553)^6 \cdot (1 - 0.553)^4 = \boxed{0.24}.$$
7. (a) Let X denote the number of students who end up in the committee. Then X is binomial with $N = 6$ and $p = 0.5$. We want to find $P(1 \leq X < 6)$. We can use the complement rule and obtain
   $$P(1 \leq X < 6) = 1 - P(X = 0) - P(X = 6).$$
   We use binomial probability:
   $$P(X = 0) = C(6,0) \cdot (0.5)^0 \cdot (0.5)^6 = 0.5^6 = 0.015625.$$
   $$P(X = 6) = C(6,6) \cdot (0.5)^6 \cdot (0.5)^0 = 0.5^6 = 0.015625.$$
   Therefore
   $$P(1 \leq X < 6) = 1 - P(X = 0) - P(X = 6) = 1 - 2 \cdot 0.015625 = 0.96875 \approx \boxed{97\%}.$$

(b) We have

$$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X = 0) - P(X = 1).$$

Using binomial probability:

$$P(X = 1) = C(6, 1) \cdot (0.5)^1 \cdot (0.5)^5 = 0.09375.$$

Therefore

$$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X = 0) - P(X = 1) = 1 - 0.015625 - 0.09375 = 0.890625 \approx \boxed{89\%}.$$

8. The possible values of X are \$0, \$5, \$10 and \$30. We have

$$P(X = 0) = P(\text{red card}) = \frac{26}{52} = 0.5.$$

$$P(X = 5) = P(\text{spade}) = \frac{13}{52} = 0.25.$$

$$P(X = 10) = P(\text{club, but not ace}) = \frac{12}{52} = 0.23.$$

$$P(X = 30) = P(\text{ace of clubs}) = \frac{1}{52} = 0.019.$$

The distribution of X is then given by

x_i	\$0	\$5	\$10	\$30
$P(X = x_i)$	0.5	0.25	0.23	0.019

The expected value of X is given by

$$E(X) = 0 \cdot 0.5 + 5 \cdot 0.25 + 10 \cdot 0.23 + 30 \cdot 0.019 = \boxed{\$4.12}.$$

9. Binomial probability: each side has probability $1/4 = 0.25$ to come up. The number of trials is $n = 3$.

(a) Let X be the number of *nun*s. Then

$$P(X \geq 1) = 1 - P(X = 0) = 1 - (1 - 0.25)^3 = \boxed{0.58}.$$

(b) Let X be the number of *nun*s. Then

$$P(X = 2) = C(3, 2) \cdot 0.25^2 \cdot (1 - 0.25) = 3 \cdot 0.25^2 \cdot 0.75 = \boxed{0.14}.$$

(c) Let X denote the number of *hei*s. Then

$$P(X = 1) = C(3, 1) \cdot 0.25 \cdot (1 - 0.25)^2 = 3 \cdot 0.25 \cdot 0.75^2 = \boxed{0.42}.$$

(d) Let X denote the number of *gimel*s. Then

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = 1 - P(X = 3) = 1 - 0.25^3 = \boxed{0.98}.$$

10. Let X denote the number of games won. Then X is binomial with $p = 18/38$. We have

$$Np \geq 10 \Longrightarrow N \cdot \frac{18}{38} \geq 10 \Longrightarrow N \geq \frac{38 \cdot 10}{18} = 21.11 \approx \boxed{21}.$$

11. Let X denote the number of question guessed correctly. Then X is binomial with $N = 10$ and $p = 0.25$. We have

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1).$$

We compute each probability using the binomial formula:

$$P(X = 0) = C(10, 0)0.25^0 \cdot 0.75^1 0 = 0.056.$$

$$P(X = 1) = C(10, 1)0.25^1 \cdot 0.75^9 = 0.1877.$$

It follows that

$$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X = 0) - P(X = 1) = 1 - 0.056 - 0.1877 = 0.7563 \approx \boxed{76\%}.$$

12. The mean is given by

$$\overline{x} = \frac{9 + 13 + 17 + 22 + 26 + 34 + 42 + 45 + 53}{9} = \boxed{29}.$$

The median is $\boxed{26}$.

13. (a) The z-score is given by $\frac{83 - 77}{5} = 1.20$. The area to the left of 1.20 is given by 0.88493, therefore the area to the right is given by $1 - 0.88493 \approx \boxed{11.5\%}$.

(b) We want the find the 10^{th} percentile. The corresponding z-score is given by $z_0 = -1.28$. It follows that

$$\frac{x_0 - 77}{5} = -1.28 \Longrightarrow x_0 = 77 - 5 \cdot 1.28 = 70.6.$$

Therefore the coldest days are $\boxed{70.6^\circ F}$ or lower.

14. (a) We want to find the area to the left of $x_0 = 0$. The z-score is given by $z_0 = \frac{0 - 0.147}{0.33} = -0.45$. The area to the left is then given by $0.32636 \approx \boxed{33\%}$.

(b) We want to find the 85^{th} percentile. From the table, we have that the z-score corresponding to the 85^{th} percentile is $z_0 = 1.04$. It follows that

$$\frac{x_0 - 0.147}{0.33} = 1.04 \Longrightarrow x_0 = 0.4902 \approx \boxed{49\%}.$$

15. (a) The z-score is given by $z_0 = \frac{48 - 55}{6} = -1.17$. The area to the left of -1.17 is then given by $0.12100 = \boxed{12.1\%}$.

(b) The z-score of 65 is given by $z_{65} = \frac{65 - 55}{6} = 1.67$. The area to the left of 1.67 is given by 0.9525. The z-score of 60 is given by $z_{60} = \frac{60 - 55}{6} = 0.83$. The area to the left of 0.83 is given by 0.79673. It follows that the area between 60 and 65 is given by $0.9525 - 0.79673 = 0.1558 \approx \boxed{15.6\%}$.

(c) The z-score corresponding to the 90^{th} percentile is $z_0 = 1.28$. Therefore

$$\frac{x_0 - 55}{6} = 1.28 \Longrightarrow x_0 = \boxed{62.68"}.$$

(d) The z-score is given by $z_0 = \dfrac{54-55}{6} = -0.17$. The area to the left of -0.17 is given by $0.43251 \approx \boxed{43\%}$.

16. The z-score is given by $\dfrac{70-72.6}{4.78} = -0.54$. The area to the left is given by 0.29460. It follows that the area to the right of 70 is given by $1-0.29460 = 0.7054 \approx \boxed{70.5\%}$.

17. (a) The z-score is $z_0 = -1.64$. The corresponding time is given by

$$\frac{x_0 - 4,313}{583} = -1.64 \Longrightarrow x_0 = 4,313 - 1.64 \cdot 583 = 3,356.88 \approx \boxed{3,357}.$$

(b) The z-score is is $z_0 = 1.28$. The actual value x_0 is given by

$$\frac{x_0 - 5,261}{807} = 1.28 \Longrightarrow x_0 = 5,261 + 1.28 \cdot 807 = 6,293.96 \approx \boxed{6,294}.$$

18. We have that the value $x_0 = 1,800$ is the 75^{th} percentile of the distribution. The corresponding z-score z_0 is given by $z_0 = 0.68$. It follows that

$$\frac{1,800 - 1,650}{\sigma} = 0.68 \Longrightarrow \sigma = \frac{150}{0.68} = \boxed{\$220.6}.$$