Finite Math, MATH 1100

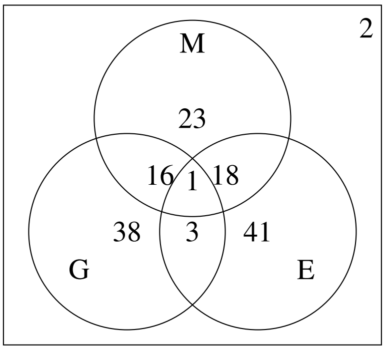
Exercises review 1 (for the first midterm)

Solutions

1. We have $A' = \{6,7,8,9,10\}$. Thus, $A' \cup B = \{2,3,4,6,7,8,9,10\}$ and $\boxed{(A' \cup B) \cap C = \{3,4,6,7,8\}}$.

2. We have $\boxed{y} = n(A \cap B) = \boxed{12}$, and $n(B) = y + z = 25 \Rightarrow \boxed{z} = 25 - 12 = \boxed{13}$. Since $n(A') = 31 = z + t$, it follows that $\boxed{t} = 31 - 13 = \boxed{18}$. Finally, $n(A' \cup B') = x + z + t = 46$, which implies $\boxed{x} = 46 - 18 - 13 = \boxed{15}$.

3. The Venn diagram of the problem is the following:



   (a) In total, $23 + 18 + 41 + 16 + 1 + 3 + 38 + 2 = \boxed{142}$ people were interviewed.

   (b) We have that $23 + 41 + 38 = \boxed{102}$ people use only one kind of tools.

4. Writing $P(E') = 1 - P(E)$ we have

$$\frac{P(E)}{1 - P(E)} = \frac{3}{2}.$$

   Set $P(E) = x$. We want to solve the equation

$$\frac{x}{1-x} = \frac{3}{2} \Longrightarrow 2x = 3 - 3x \Longrightarrow 5x = 3 \Longrightarrow x = \frac{3}{5}.$$

   Therefore $\boxed{P(E) = \frac{3}{5}}$ and the probability of losing is $\boxed{P(E') = \frac{2}{5}}$.

5. (a) If A and B are mutually exclusive, $P(A \cap B) = 0$. Therefore $P(B) = 1 - 0.5 - 0.3 = \boxed{0.2}$.

   (b) Since $P(A' \cap B') = 0.3$, then $P(A \cup B) = 1 - 0.3 = 0.7$. Then (using independence, we have $P(A \cap B) = P(A) \times P(B)$)

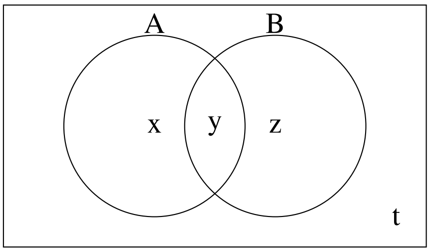
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = 0.5 + P(B) - P(A) \times P(B)$$

$$0.2 = P(B) - 0.5 \times P(B)$$

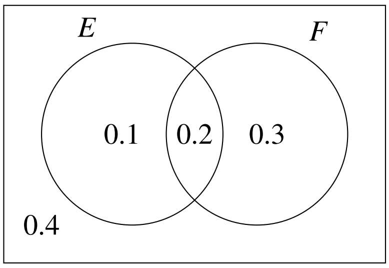
$$0.2 = 0.5 \times P(B) \Longrightarrow P(B) = \boxed{0.4}.$$

6. We draw a Venn diagram as follows (here x, y, z, t represent probabilities).



   Since $P(A \cup B) = 0.7$, then $t = 1 - 0.7 = 0.3$. We have $P(A \cup B') = x + y + t = 0.9$, thus $x + y = 0.6$ and therefore $\boxed{P(A) = 0.6}$.

7. The Venn diagram for the events is given by:



   (a) $P(E' \cup F') = P(E') + P(F') - P(E' \cap F') = 0.7 + 0.5 - 0.4 = \boxed{0.8}$.

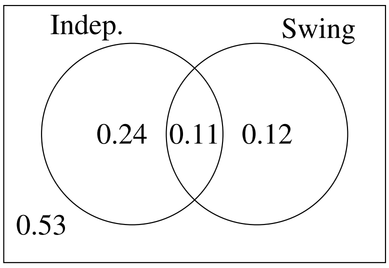
   (b) $P(E' \cap F) = \boxed{0.3}$.

   (c) We have
   $$P(E/F') = \frac{P(E \cap F')}{P(F')} = \frac{0.1}{0.5} = \boxed{0.2}.$$

   (d) We have
   $$P(E'/F') = \frac{P(E' \cap F')}{P(F')} = \frac{0.4}{0.5} = \boxed{0.8}.$$

8. (a) No, since their intersection is not empty (11% identify as both).

   (b) The Venn diagram for the events is given by:



   (c) $P(\text{Indep.} \cap \text{swing}') = \boxed{0.24}$.

(d) $P(\text{Indep.} \cup \text{swing}) = 0.24 + 0.11 + 0.12 = \boxed{0.47}$.

(e) $P(\text{Indep.}' \cap \text{swing}') = \boxed{0.53}$.

(f) We have

$$P(\text{Indep.}) \cdot P(\text{swing}) = 0.35 \cdot 0.23 = 0.0805 \neq 0.11 = P(\text{Indep.} \cap \text{swing}),$$

and therefore they are not independent.

9. (a) Let A_i denote the answer of question 1. We have, using the fact the the answers are independent,

$$P(A_1 = F \cap A_2 = F \cap A_3 = F \cap A_4 = F \cap A_5 = T) =$$

$$P(A_1 = F) \cdot P(A_2 = F) \cdot P(A_3 = F) \cdot P(A_4 = F) \cdot P(A_5 = T) = (0.75)^4 \cdot 0.25 = \boxed{0.079}.$$

(b) Similarly, we have

$$P(A_1 = T \cap A_2 = T \cap A_3 = T \cap A_4 = T \cap A_5 = T) = 0.25^5 = \boxed{0.00098}.$$

(c) We use the complement:

$$P(\text{at least one right}) = 1 - P(\text{all wrong}) = 1 - 0.75^5 = \boxed{0.76}.$$

10. $(0.98)^9 \cdot 0.02 = \boxed{0.0167}$.

11. Let E be the event: "the sum is at least 9", and F the event: "at least one die shows a 5". We want to find $P(E|F)$. We have $E \cap F = \{(4,5),(5,4),(5,5),(6,5),(5,6)\}$, $F = \{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5),(5,1),(5,2),(5,3),(5,4),(5,6)\}$. It follows that

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{5/36}{11/36} = \boxed{\frac{5}{11}}.$$

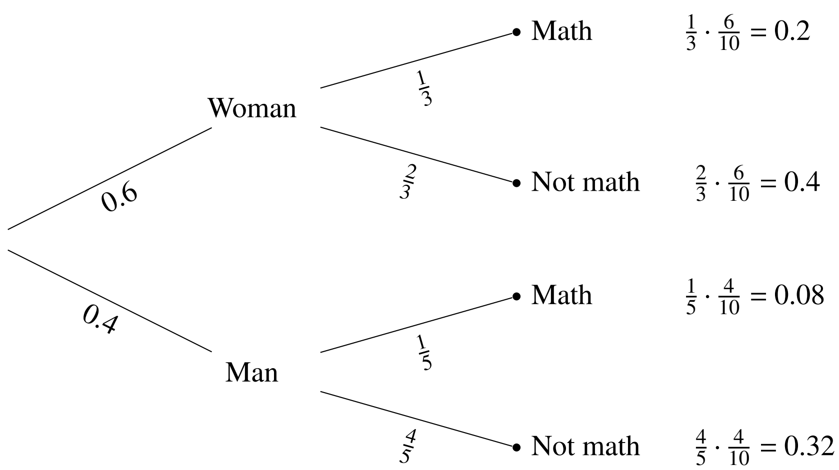
12. We have

$$P(\text{king}|\text{face}) = \frac{P(\text{king} \cap \text{face})}{P(\text{face})} = \frac{P(\text{king})}{P(\text{face})} = \frac{4/52}{12/52} = \boxed{\frac{1}{3}}.$$

13. We use the complement rule:

$$P(\text{al least one leggings}) = 1 - P(\text{no leggings}) = 1 - \frac{19}{24} \cdot \frac{18}{23} \cdot \frac{17}{22} = \boxed{0.52}.$$

14. We construct the tree diagram for the events.



(a) i. $P(\text{woman} \cap \text{math}) = \boxed{0.2}$.

ii. $P(\text{man} \cap \text{math}) = \boxed{0.08}$.

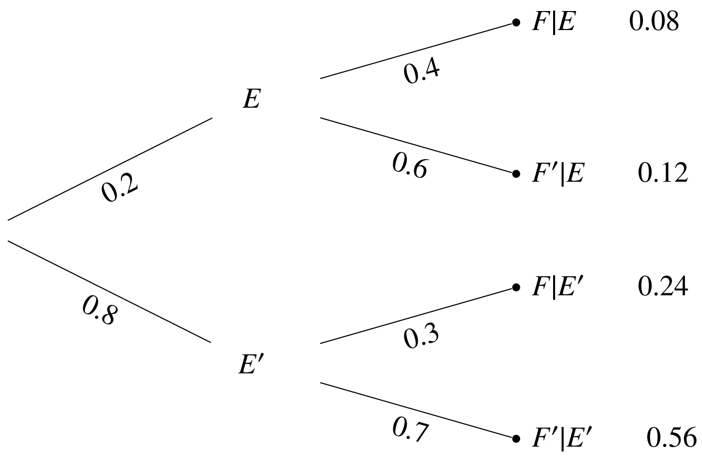
iii. $P(\text{math}) = 0.2 + 0.08 = \boxed{0.28}$.

iv. $P(\text{not math}) = 0.4 + 0.32 = \boxed{0.72}$.

(b) We have

$$P(\text{woman}|\text{math}) = \frac{P(\text{woman} \cap \text{math})}{P(\text{math})} = \frac{0.2}{0.28} \approx \boxed{71.4\%}.$$

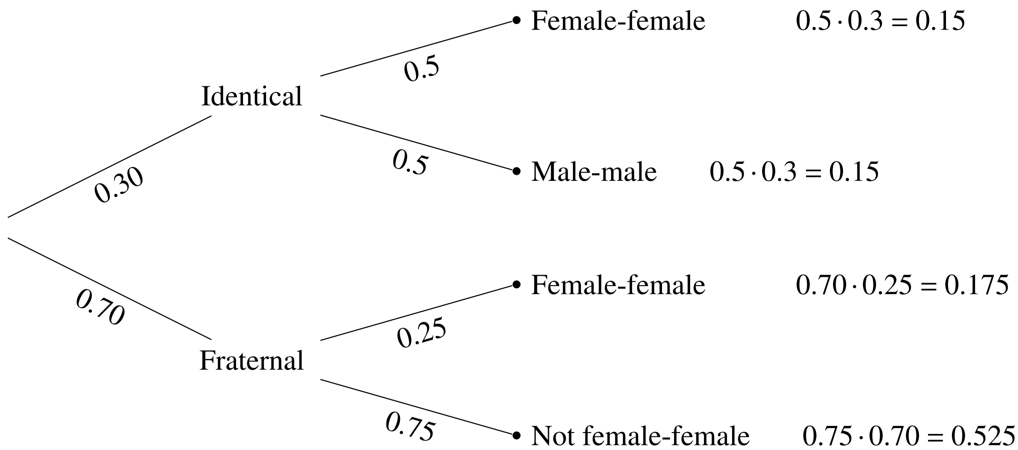
15. The tree diagram of the problem is given by



We have

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.08}{0.08 + 0.24} = \boxed{0.25}.$$

16. A tree diagram for the problem is given by



We have

$$P(\text{identical}|\text{female-female}) = \frac{P(\text{identical} \cap \text{female-female})}{P(\text{female-female})} = \frac{0.15}{0.15 + 0.175} = \boxed{0.46}.$$