

8.4 Binomial Probability

Example/Discussion Problems

1. Three marbles are randomly selected from a jar containing 6 red and 8 blue marbles.
 - (a) What is the probability that exactly two marbles are red?
 - i. Use combinations and the multiplication principle.
 - ii. Use a tree diagram and the product rule.
 - (b) What if instead the jar contains 6,000,000 red marbles and 8,000,000 blue marbles?
 - (c) What if the marbles are replaced back in the jar after each selection?

Binomial Experiment

1. The same experiment is repeated a fixed number of times.
2. There are only two possible outcomes, success and failure.
3. The repeated trials are independent, so that the probability of success remains the same for each trial.

Binomial Probability

If p is the probability of success in a single trial of a binomial experiment, the probability of x successes and $n - x$ failures in n independent repeated trials of the experiment, known as **binomial probability**, is

$$P(x \text{ successes in } n \text{ trials}) = C(n, x) \cdot p^x \cdot (1 - p)^{n-x}.$$

2. A basketball player makes 80% of their free throws. If this player takes 8 free throws in a game, what is the probability that they make at least 6 of them?
3. An office printer is broken in such a way that every time it prints a page, there is a 2% chance that a paper jam will occur.
 - (a) What is the probability that 10 pages are printed without a single paper jam?
 - (b) What is the probability that 100 pages are printed with no more than 2 paper jams?
4. A hospital receives $1/5$ of its flu vaccine shipments from Company X and the remainder of its shipments from other companies. Each shipment contains a very large number of vaccine vials. For Company X's shipments, 10% of the vials are ineffective. For every other company, 2% of the vials are ineffective. The hospital tests 30 randomly selected vials from a shipment and finds that one vial is ineffective. What is the probability that this shipment came from Company X?

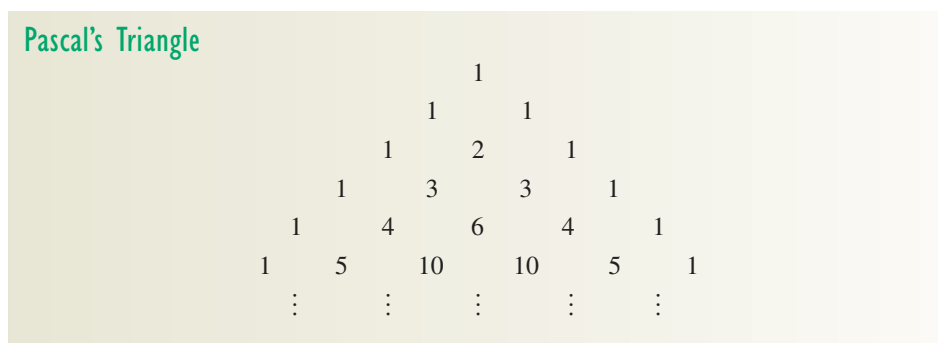


Figure 1: $C(n, r) + C(n, r + 1) = C(n + 1, r + 1)$.

1. Three marbles are randomly selected from a jar containing 6 red and 8 blue marbles.

(a) What is the probability that exactly two marbles are red?

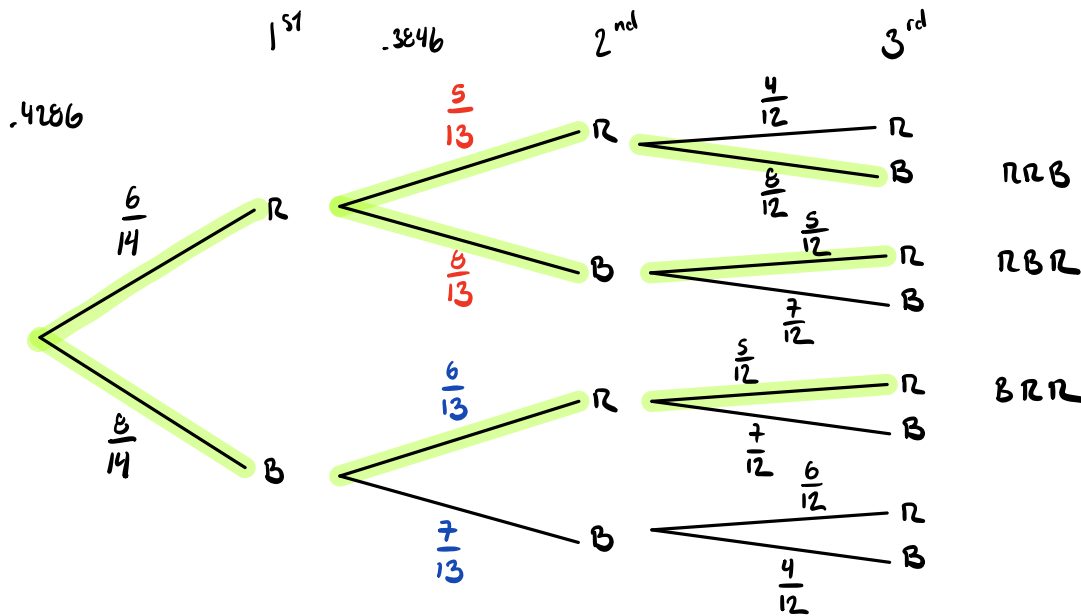
- Use combinations and the multiplication principle.
- Use a tree diagram and the product rule.

(a) $A = 2 \text{ red}, 1 \text{ blue}$

CHOOSE 2 RED FROM 6 RED CHOOSE 1 BLUE FROM 8 BLUE
 ↓ 6 RED ↓ 8 BLUE

$$P(A) = \frac{n(A)}{n(S)} = \frac{C(6, 2) \times C(8, 1)}{C(14, 3)} = \frac{\frac{6!}{2!4!} \times \frac{8!}{1!7!}}{\frac{14!}{3!11!}}$$

$$= \frac{15 \times 8}{364} = .3297$$



$$P(2r, 1b) = P(RRB) + P(RBR) + P(BRR)$$

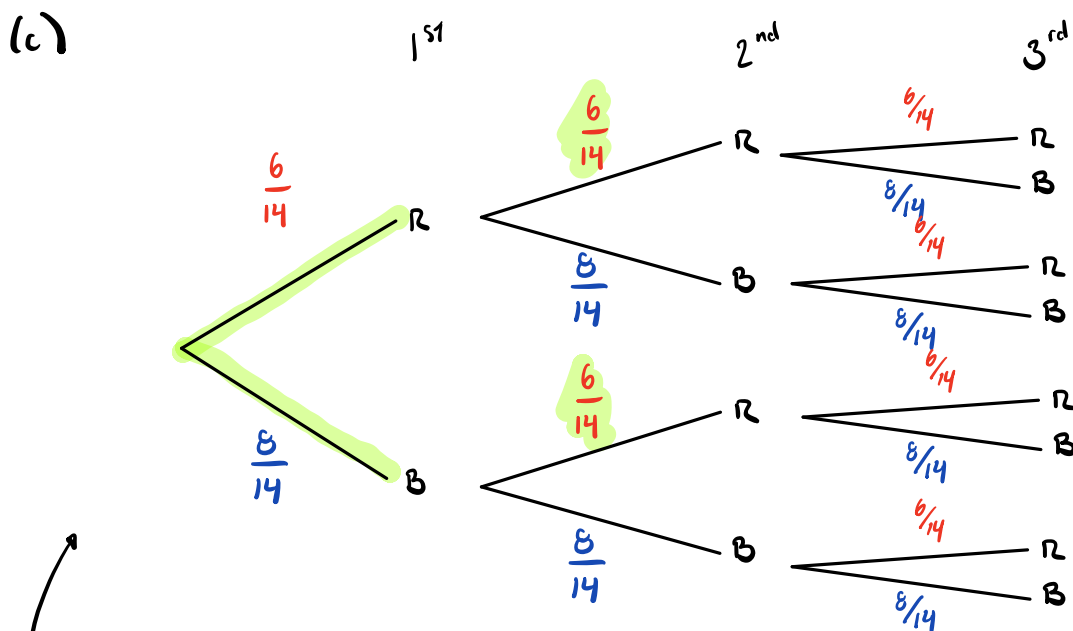
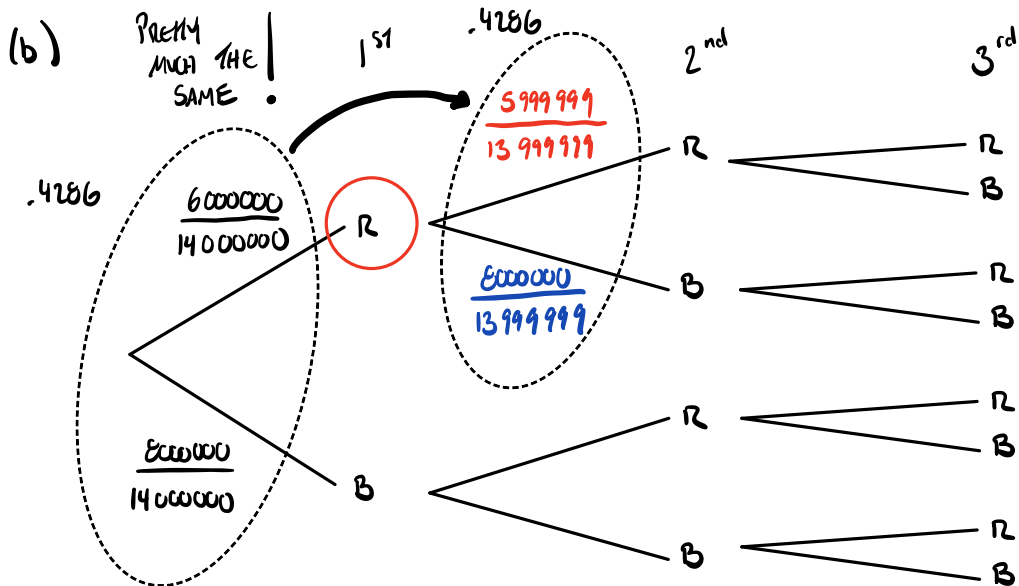
$$= \frac{6}{14} \cdot \frac{5}{13} \cdot \frac{8}{12} + \frac{6}{14} \cdot \frac{6}{13} \cdot \frac{5}{12} + \frac{8}{14} \cdot \frac{6}{13} \cdot \frac{5}{12}$$

Product Rule

$$= .3297$$

(b) What if instead the jar contains 6,000,000 red marbles and 8,000,000 blue marbles?

(c) What if the marbles are replaced back in the jar after each selection?



NOW, THIS EXPERIMENT (SELECTING 3 MARBLES, WITH REPLACEMENT)

IS COMPOSED OF REPEATING THE SAME TRIAL,

EACH WITH EXACTLY 2 POSSIBLE OUTCOMES (RED & BLUE),

WITH SAME PROBABILITIES ($\frac{6}{14}$, $\frac{8}{14}$)

Success & Failure

Binomial Experiment Trial

1. The same experiment is repeated a fixed number of times.
2. There are only two possible outcomes, success and failure.
3. The repeated trials are independent, so that the probability of success remains the same for each trial.

p $q = 1 - p$

Binomial Probability

If p is the probability of success in a single trial of a binomial experiment, the probability of x successes and $n - x$ failures in n independent repeated trials of the experiment, known as **binomial probability**, is

$$P(x \text{ successes in } n \text{ trials}) = C(n, x) \cdot p^x \cdot (1 - p)^{n-x}.$$

$$p = P(\text{SUCCESS}) \text{ FOR ANY TRIAL}$$

$$q = P(\text{FAILURE}) \text{ FOR ANY TRIAL}$$

$$= 1 - P(\text{SUCCESS})$$

$$= 1 - p$$

ex.

Three marbles are randomly selected from a jar containing 6 red and 8 blue marbles.

Each marble is replaced back in the jar after it is selected.

Find the probability that exactly 2 red marbles are selected.

NOTE: IT IS POSSIBLE
TO CHOOSE SAME
MARBLE TWICE!

BINOMIAL EXPERIMENT:

3 TRIALS

EACH TRIAL HAS 2 POSSIBLE OUTCOMES

SUCCESS : RED MARBLE

FAILURE : BLUE MARBLE

$$p = P(\text{SUCCESS}) = \frac{6}{14}$$

$$q = P(\text{FAILURE}) = 1 - p = \frac{8}{14}$$

} FOR EACH TRIAL (INDEPENDENT)

3 TRIALS

$$P(2 \text{ SUCCESSSES, } 1 \text{ FAILURE}) = P(2R, 1B)$$

WAYS TO ORDER 2 SUCCESSSES (R)
& 1 FAILURE (B)

$$= P(RRB) + P(RBR) + P(BRR)$$

WAYS TO ORDER 2 SUCCESSSES (R)
& 1 FAILURE (B)

$$= \left(\frac{6}{14}\right)\left(\frac{6}{14}\right)\left(\frac{8}{14}\right) + \left(\frac{6}{14}\right)\left(\frac{8}{14}\right)\left(\frac{6}{14}\right) + \left(\frac{8}{14}\right)\left(\frac{6}{14}\right)\left(\frac{6}{14}\right)$$

$$C(3, 2) = 3$$

$$= 3 \left(\frac{6}{14}\right)^2 \left(\frac{8}{14}\right)^1 = 3p^2q$$

CHOOSE 2 TRIALS
FOR SUCCESSSES

$$= .3149$$

$$P(\underbrace{2 \text{ success}}_{x = \# \text{ successes}} \text{ in } \underbrace{3 \text{ trials}}_{n = \# \text{ trials}}) = C(3, 2) p^2 q^1$$

Binomial Probability

If p is the probability of success in a single trial of a binomial experiment, the probability of x successes and $n - x$ failures in n independent repeated trials of the experiment, known as **binomial probability**, is

$$P(x \text{ successes in } n \text{ trials}) = C(n, x) \cdot p^x \cdot \underbrace{(1 - p)^{n-x}}_q$$

n TRIALS,
 x SUCCESSSES
 $\#$ FAILURES: $n - x$

2. A basketball player makes 80% of their free throws. If this player takes 8 free throws in a game, what is the probability that they make at least 6 of them?

Binomial Experiment!

TRIALS $n = 8$
 SUCCESS $p = .8$
 FAILURE $q = .2$
 $x = \#$ SUCCESSSES

Binomial Experiment ^{TRIAL}

1. The same experiment is repeated a fixed number of times.
2. There are only two possible outcomes, success and failure.
3. The repeated trials are independent, so that the probability of success remains the same for each trial.

$$P(x \text{ successses in } n \text{ trials}) = C(n, x) p^x q^{n-x}$$

$$\begin{aligned} P(x \geq 6) &= P(6 \text{ successes in } 8 \text{ trials}) + P(7 \text{ successes in } 8 \text{ trials}) + P(8 \text{ successes in } 8 \text{ trials}) \\ &\quad \uparrow \\ &\quad x \leq n \\ &= C(8, 6) (.8)^6 (.2)^2 + C(8, 7) (.8)^7 (.2)^1 + C(8, 8) (.8)^8 (.2)^0 \\ &= .7969 \end{aligned}$$

3. An office printer is broken in such a way that every time it prints a page, there is a 2% chance that a paper jam will occur.

0 paper jams

- (a) What is the probability that 10 pages are printed without a single paper jam?
 (b) What is the probability that 100 pages are printed with no more than 2 paper jams?

$$\text{BINOMIAL PROBABILITY: } P(x \text{ successes in } n \text{ trials}) = C(n, x) p^x q^{n-x}$$

(a) BINOMIAL EXPERIMENT # trials $n = 10$; $x = \# \text{ successes}$

SUCCESS = JAM $\rightarrow p = .02$

FAILURE = NO JAM $\rightarrow q = .98$

$$P(x=0) = \underbrace{C(10, 0)}_1 \underbrace{(.02)^0}_{1} (.98)^{10} = .98^{10} \approx .8171$$

trials $n = 10$; $x = \# \text{ successes}$

SUCCESS = NO JAM $\rightarrow q = .98$

FAILURE = JAM $\rightarrow p = .02$

$$P(x=10) = \underbrace{C(10, 10)}_1 \underbrace{(.98)^{10}}_1 (.02)^0 = .98^{10} \approx .8171$$

(b) BINOMIAL EXPERIMENT $n = 100$ SUCCESS = NO JAM $\rightarrow p = .98$
 FAILURE = JAM $\rightarrow q = .02$

$x = \# \text{ successes} = \# \text{ NO JAMS } (\# \text{ successfully printed})$

0 JAMS $\rightarrow$ 0 FAILURES $\rightarrow$ 100 SUCCESSSES $\rightarrow x = 100$
 1 JAM $\rightarrow$ 1 FAILURES $\rightarrow$ 99 SUCCESSSES $\rightarrow x = 99$
 2 JAMS $\rightarrow$ 2 FAILURES $\rightarrow$ 98 SUCCESSSES $\rightarrow x = 98$

$$P(\# \text{ FAILURES} \leq 2) = P(\# \text{ SUCCESSSES} \geq 98)$$

$$= P(x=98) + P(x=99) + P(x=100)$$

$$= C(100, 98) (.98)^{98} (.02)^2 + C(100, 99) (.98)^{99} (.02)^1 + C(100, 100) (.98)^{100} (.02)^0$$

$$= .6767$$

BINOMIAL PROBABILITY : $P(x \text{ successes in } n \text{ trials}) = C(n, x) p^x q^{n-x}$