

EXAM 1: WEDNESDAY 6/9 2nd HALF ~ 10:30 AM EDT

Formula Sheet

$$\text{Addition Rule } n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\text{Product Rule } P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$

$$\text{LOTP: } P(A) = P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + \dots + P(S_n)P(A|S_n)$$

$$\text{Bayes' Rule: } P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$

- CAMERAS ON (SHOW YOUR DESK AREA)
- MICS OFF
- QUESTIONS - ZOOM CHAT (PRIVATELY)
- UPLOAD YOUR SCANNED EXAM TO GOOGLE DRIVE FOLDER BEFORE Log out.

LECTURE RECORDINGS: BLACKBOARD

8.1 The Multiplication principle and Permutations

Example/Discussion Problems

1. A sandwich shop lets customers create their own sandwich by repeatedly making choices from a selection of options as follows.
 - (a) Choose a type of bread: white, wheat, rye, sourdough.
 - (b) Choose a meat: turkey, beef, tuna.
 - (c) Choose a cheese: Cheddar, none.

How many different sandwiches can one order at this shop?

Multiplication Principle

Suppose n choices must be made, with

m_1 ways to make choice 1,

m_2 ways to make choice 2,

and so on, with

m_n ways to make choice n .

Then there are

$$m_1 \cdot m_2 \cdot \cdots \cdot m_n$$

different ways to make the entire sequence of choices.

2. How many distinct 10 digit phone numbers exist? Note that neither the 1st nor 4th digit of a 10-digit phone number can be 0 or 1?
3. How many distinct 7 character license plates exist if the first 3 characters must be letters, and the last 4 characters must be digits?
4. How many distinct 7 character license plates exist if the first 3 characters must be letters, and the last 4 characters must be digits, and characters that appear next to each other must be different?
5. How many distinct 7 character license plates exist if the first 3 characters must be letters, and the last 4 characters must be digits, and no character can appear more than once?
6. How many ways are there for a club with 15 members to choose one president, one vice-president, and one treasurer?
7. A graphic designer has 256 colors from which to choose one color as a background, one color for large type, and one color for small type. How many ways can this be done? (Do the choices need to be different?)

Factorial Notation

1, 2, 3, ...

For any natural number n ,

$$n! = n(n-1)(n-2) \cdots (3)(2)(1).$$

Also, by definition,

$$0! = 1.$$

"n FACTORIAL"

$$e.g. \quad 3! = 3 \times 2 \times 1 = 6$$

$$4! = 4 \times 3 \times 2 \times 1$$

$$= 4 \times 3! = 24$$

$$5! = 5 \times 4! = 120$$

$$(0^0 = 1) \quad \leftarrow 0 \text{ is SPECIAL!}$$

$$\text{FACT: } n! = n \times (n-1)!$$

 $P(11, 11)$

- 8. A mail deliverer needs to deliver packages to 11 different addresses. In how many different orders can they deliver the packages?

$$11! = \frac{11!}{(11-11)!} = \frac{11!}{0!} \quad \checkmark$$

 $P(8, 3)$

- 9. How many ways are there to select 3 books from a collection of 8 books and arrange them on a shelf from left to right?

$$8 \times 7 \times 6 = \frac{8!}{5!} = \frac{8!}{(8-3)!}$$

PermutationsIf $P(n, r)$ (where $r \leq n$) is the number of permutations of n elements taken r at a time, then

$$P(n, r) = \frac{n!}{(n-r)!}$$

WAYS OF

SELECTING AND ARRANGING/ORDERING

 r OBJECTS TAKEN FROM A COLLECTIONOF n OBJECTS. **$P(n, n)$** The number of permutations of a set with n elements is $n!$; that is $P(n, n) = n!$.

10. There are 8 democratic candidates running for mayor of NYC. On June 22nd, registered democrats will go to the polls and rank their top 5 choices for mayor. How many ways can registered democrats vote in the mayoral primary?
11. How many distinct ways can all of the letters in the following words be arranged?
- SWIMMING
 - BOOKKEEPER
 - MISSISSIPPI

If the n objects in a permutation are not all distinguishable—that is, if there are n_1 of type 1, n_2 of type 2, and so on for r different types, then the number of **distinguishable permutations** is

$$\frac{n!}{n_1! n_2! \cdots n_r!}$$

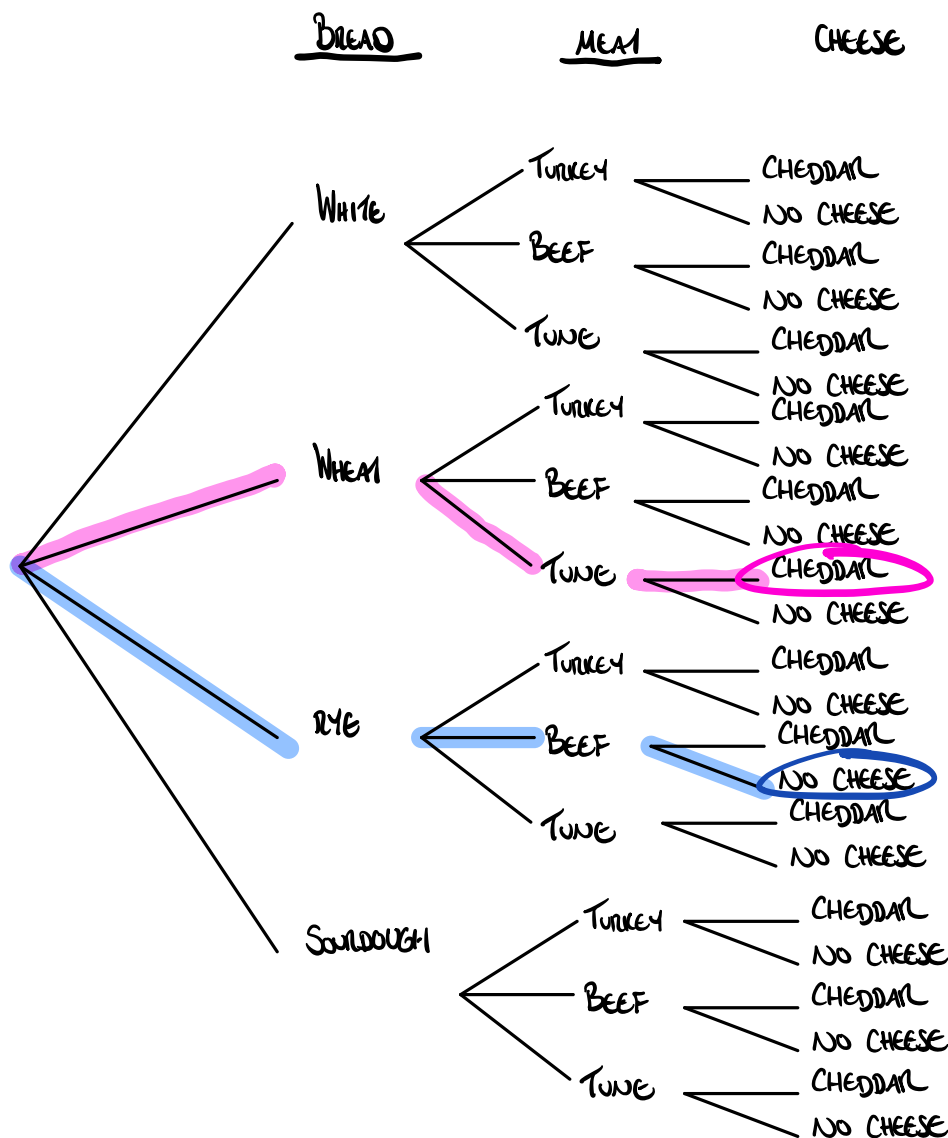
1. A sandwich shop lets customers create their own sandwich by repeatedly making choices from a selection of options as follows.

- (a) Choose a type of bread: white, wheat, rye, sourdough.
- (b) Choose a meat: turkey, beef, tuna.
- (c) Choose a cheese: Cheddar, none.

How many different sandwiches can one order at this shop?

- 1. White, Turkey, Cheddar
- 2. Wheat, Beef, No Cheese
- 3 ...

Get organized :



$$P(E) = \frac{n(E)}{n(S)}$$

counting
"Combinatorics"

4 Groups of
3 Groups of
2

$$\Rightarrow 4 \times 3 \times 2 = 24 \text{ SANDWICHES}$$

RYE, BEEF, NO CHEESE

1:1 CORRESPONDANCE BETWEEN PATHS THROUGH TREE & SANDWICHES

2. How many distinct 10 digit phone numbers exist? Note that neither the 1st nor 4th digit of a 10-digit phone number can be 0 or 1?
3. How many distinct 7 character license plates exist if the first 3 characters must be letters, and the last 4 characters must be digits?
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2.

CHOICES:

$$\begin{array}{cccccccccc} \underline{8} & \underline{10} & \underline{10} & \underline{8} & \underline{10} & \underline{10} & \underline{10} & \underline{10} & \underline{10} & \underline{10} \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array}$$

MULT. PRINCIPLE: $8^2 \times 10^8 = 64,000,000,000$ 6.4 Billion!

3.

CHOICES
OPTIONS

$$\underline{26} \times \underline{26} \times \underline{26} \times \underline{10} \times \underline{10} \times \underline{10} \times \underline{10} = 26^3 \times 10^4 = 175,760,000$$

MULT. PRINC.

4.

CHOICES
OPTIONS

DIFFERENT
↖

$$\underline{26} \times \underline{25} \times \underline{25} \times \underline{10} \times \underline{9} \times \underline{9} \times \underline{9} = 26 \times 25^2 \times 10 \times 9^3$$

MULT. PRINC.

=

5. How many distinct 7 character license plates exist if the first 3 characters must be letters, and the last 4 characters must be digits, and no character can appear more than once?
6. How many ways are there for a club with 15 members to choose one president, one vice-president, and one treasurer?

5.

$$\underline{26} \times \underline{25} \times \underline{24} \times \underline{10} \times \underline{9} \times \underline{8} \times \underline{7} = 78,624,000$$

6.

$$\frac{15}{P} \times \frac{14}{VP} \times \frac{13}{T} = 2,730$$

8. A mail deliverer needs to deliver packages to 11 different addresses. In how many different orders can they deliver the packages?
9. How many ways are there to select 3 books from a collection of 8 books and arrange them on a shelf from left to right?

8.

$$\underline{11} \times \underline{10} \times \underline{9} \times \underline{8} \times \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 11!$$

MULT. PRINC.

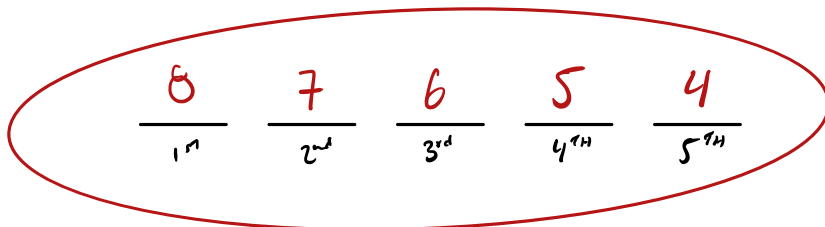
$$= 39916800$$

9.

$$\underline{8} \times \underline{7} \times \underline{6} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{8!}{5!}$$

$$= \frac{8!}{(8-3)!}$$

10. There are 8 democratic candidates running for mayor of NYC. On June 22nd, registered democrats will go to the polls and rank their top 5 choices for mayor. How many ways can registered democrats vote in the mayoral primary?



8 objects, CHOOSE & ARRANGE 5

$$P(8, 5) = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}}$$

11. How many distinct ways can all of the letters in the following words be arranged?

- (a) SWIMMING
(b) BOOKKEEPER
(c) MISSISSIPPI

abcdefgh $\rightarrow 8! \quad P(8, 8)$

(a) WGINM₁IM₂S $\leftrightarrow$ WGINM₂IM₁S SHOULD ONLY BE COUNTED ONCE!

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M - cousins

EVERY WORD HAS AN M-COUSIN.

TO ONLY COUNT M-COUSINS ONCE, WE DIVIDE  $P(8, 8) = 8!$  BY 2.

SIMILARLY, TO ONLY COUNT I-COUSINS ONCE, WE AGAIN DIVIDE BY 2.

SWIMMING  $\rightarrow$  SWIMMING

$$\frac{8!}{2 \times 2} = \frac{8!}{2! \cdot 2!}$$

(b) BOOKKEEPER

$$\frac{10!}{2! \cdot 2! \cdot 3!}$$

EACH WORD HAS 6 EQUIVALENT  
ARRANGEMENTS OBTAINED BY  
PERMUTING E'S