§7.5 Conditional Probability & Independent Events

Conditional Probability:

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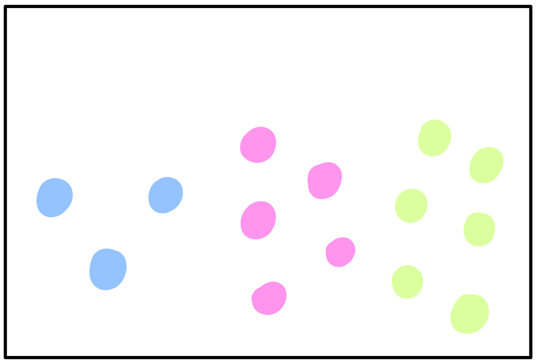


Experiment: Select one marble.

Events: A = Blue
B = Pink
C = Green

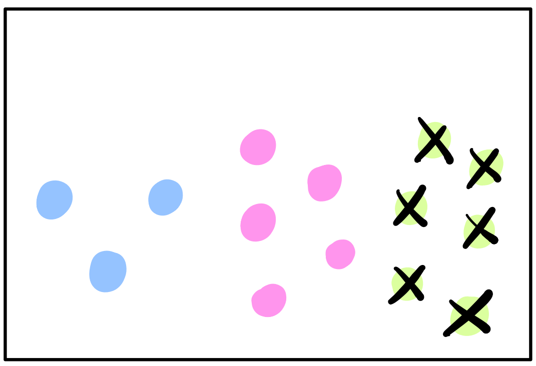
$P(A) = \frac{3}{14}$

Suppose some information about the outcome of this experiment is obtained.

If you know the selected marble is not green (C')
now what is probability the selected marble is blue (A)?

$\frac{3}{8}$

Sample space is smaller



S = { all possible outcomes }

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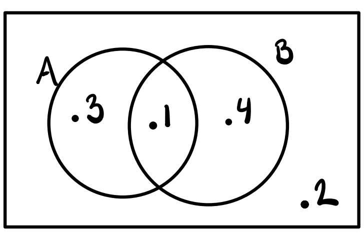
ex. Suppose an experiment that can result in events A & B:



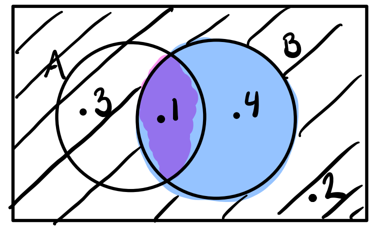
$P(A) = P(A \cap B') + P(A \cap B)$
$= .3 + .1$
$= .4$

$P(B) = .5$

Conditional Probability: Given knowledge that B occurs, what is the probability that A occurs

NOTATION: $P(A \mid B) = \frac{.1}{.5} = .2$

"A GIVEN B"



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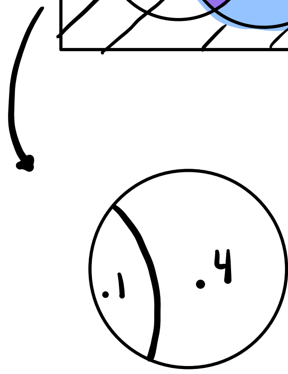

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ex. EXPERIMENT: ROLL 2 DICE.

FIND THE PROBABILITY THAT AT LEAST ONE 1 IS ROLLED, GIVEN THAT THE SUM IS LESS THAN OR EQUAL TO 4.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$\longrightarrow$

	1	2	3	4	5	6
1	2	3	4			
2	3	4				
3	4					
4						
5						
6						

EVENTS: A: AT LEAST ONE 1 IS ROLLED
B: SUM ≤ 4

$$P(A \mid B) = \frac{n(A \cap B)}{n(B)} = \boxed{\frac{5}{6}}$$

$$= \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{P(A \cap B)}{P(B)} = \frac{5/36}{6/36}$$

GIVEN 2 EVENTS A, B, THE CONDITIONAL PROBABILITY OF A GIVEN B IS

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

Natural Science *A scientist wishes to determine whether there is any dependence between color blindness (C) and deafness (D). Use the probabilities in the table to answer Exercises 41 and 42.*

	D	*D'*	Total
C	.0004	.0796	.0800
C'	.0046	.9154	.9200
Total	.0050	.9950	1.0000

41. Find $P(C)$, $P(D)$, $P(C \cap D)$, and $P(C|D)$. Does $P(C) = P(C|D)$? What does that result imply regarding independence of C and D?

42. Find $P(D')$, $P(C)$, $P(D' \cap C)$, and $P(D'|C)$. Does $P(D') = P(D'|C)$? What does that result imply regarding independence of D' and C?

41. $P(C) = .0800$

$P(D) = .0050$

$P(C \cap D) = .0004$

$$P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{.0004}{.0050} = .08$$

INDEPENDENT EVENTS:

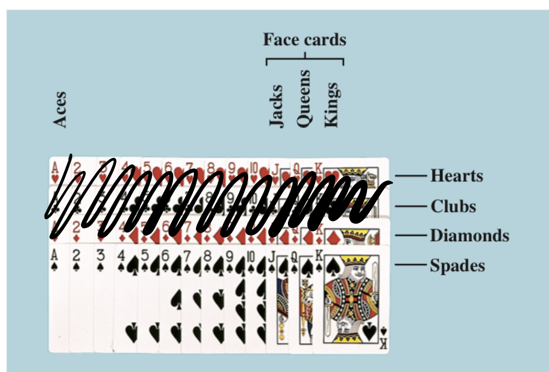
Def: 2 EVENTS A, B ARE INDEPENDENT IF $P(A|B) = P(A)$.

KNOWLEDGE OF B OCCURRING DOES NOT CHANGE THE LIKELIHOOD OF A OCCURRING.

ex. EXPERIMENT: SELECT ONE CARD FROM STAND. DECK OF 52 CARDS.

EVENTS: A = SELECT A KING

B = SELECT A SPADE ♠.



$P(A) = \frac{4}{52} = \frac{1}{13}$

$P(B) = \frac{13}{52} = \frac{1}{4}$

SAME!

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/52}{13/52} = \frac{1}{13}$$

$\therefore$ EVENTS A & B ARE INDEPENDENT.

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{1/52}{4/52} = \frac{1}{4}$$

EQUIVALENT DEFINITIONS FOR A, B INDEPENDENT:

- $P(A|B) = P(A)$
- $P(B|A) = P(B)$
- $P(A \cap B) = P(A)P(B)$

PRODUCT RULE FOR PROBABILITY:

NOTE: $P(B) \times P(A|B) = \frac{P(A \cap B)}{\cancel{P(B)}} \times \cancel{P(B)}$

$$P(B)P(A|B) = P(A \cap B)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A)P(B|A) = P(A \cap B)$$

PRODUCT RULE FOR PROBABILITY:

$$\boxed{P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)}$$

ALWAYS TRUE

IN THE SPECIAL CASE THAT A & B ARE INDEPENDENT,

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$

$$\underbrace{P(A \cap B) = P(A)P(B) = P(B)P(A)}$$

ONE DEFINITION OF A & B BEING INDEPENDENT EVENTS.

ex. SUPPOSE A BASEBALL TEAM IS HEADING INTO PLAYOFFS.

IF THEY PLAY TEAM A, THE PROBABILITY OF WINNING IS .8 — $P(W|A) = .8$

IF THEY PLAY TEAM B, THE PROBABILITY OF WINNING IS .5 — $P(W|B) = .5$

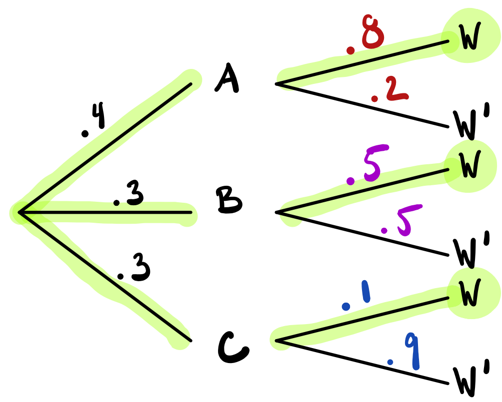
IF THEY PLAY TEAM C, THE PROBABILITY OF WINNING IS .1 — $P(W|C) = .1$

SUPPOSE THE PROBABILITY THIS TEAM PLAYS TEAM A IS .4

SUPPOSE THE PROBABILITY THIS TEAM PLAYS TEAM B IS .3

SUPPOSE THE PROBABILITY THIS TEAM PLAYS TEAM C IS .3

ADD TO 1.



WHAT IS THE PROB THAT THIS TEAM WINS AGAINST TEAM A?

$$P(A \cap W) = P(A)P(W|A) = (.4)(.8) = .32$$

$$P(C \cap W') = P(C)P(W'|C) = (.3)(.9) = .27$$

Q WHAT IS THE PROB. THIS TEAM WINS?

$$P(W) = P\big((A \cap W) \cup (B \cap W) \cup (C \cap W)\big)$$

DISJOINT EVENTS

NO OVERLAP

LoTP

$$= P(A \cap W) + P(B \cap W) + P(C \cap W)$$

$$= \boxed{P(A)P(W|A) + P(B)P(W|B) + P(C)P(W|C)}$$

$$= (.4)(.8) + (.3)(.5) + (.3)(.1)$$

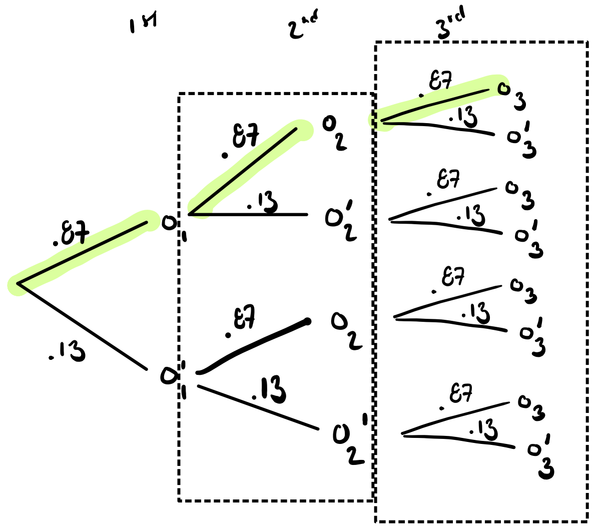
$$= .32 + .15 + .03$$

$$= .5$$

58. Business According to data from the U.S. Department of Transportation, Delta Airlines was on time approximately 87% of the time in 2012. Use this information, and assume that the event that a given flight takes place on time is independent of the event that another flight is on time to answer the following questions

(a) Elisabeta Gueyara plans to visit her company's branch offices; her journey requires 3 separate flights on Delta Airlines. What is the probability that all of these flights will be on time?

(b) How reasonable do you believe it is to suppose the independence of being on time from flight to flight?



Prob that 2nd & 3rd flights are on time do not depend on whether previous flights were on time.
Independent.

Let O_i = i^{th} flight on time

Product Rule for Prob.

$$P(O_1 \cap O_2 \cap O_3)$$
$$= P(O_1)P(O_2)P(O_3)$$
$$= (.87)^3$$
$$= .6585$$

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ex. Suppose your neighbor has 2 kids.
Suppose you meet one of those kids, Billy (boy).
What is probability Billy has a sister?

Given that at least one child is a boy

S

	1st	
2nd	BB	BG
	GB	~~GG~~

3 possibilities all equally likely.

$$P(\text{at least one girl} \mid \text{not 2 girls}) = \frac{2}{3}$$