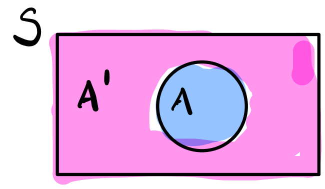
§7.4 Basic Concepts of Probability

$\boxed{\text{Addition Rule for Probability: } P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

Note: $P(S) = \dfrac{n(S)}{n(S)} = 1$



Given any event $A \subseteq S$, $A \cup A' = S$

$\therefore P(A \cup A') = P(S) = 1$

$1 = P(A \cup A') = P(A) + P(A') - P(A \cap A')$

$= P(A) + P(A') - \underbrace{P(\emptyset)}_{0}$

$\emptyset = (A \cap B) \cap (A \cap B')$

$A = (A \cap B) \cup (A \cap B')$

$P(A) = P((A \cap B) \cup (A \cap B'))$

$= P(A \cap B) + P(A \cap B') - P(\emptyset)$

Complement Rule: $P(A) + P(A') = 1$

Equivalently: $P(A') = 1 - P(A)$

Example 5 If two fair dice are rolled, find the probability that the sum of the numbers showing is greater than 3.

Sample Space:

Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Let A = sum > 3.

Find $P(A)$.

$P(A) + P(A') = 1$

$P(A) = 1 - P(A') = 1 - \dfrac{3}{36} = \dfrac{33}{36}$

$= .9167$

Business *Data from the 2012 General Social Survey can allow us to estimate how much part-time and full-time employees work per week. Use the following table to find the probabilities of the events in Exercises 55–58. (Data from: www3.norc.org/gss+website.)*

		Hours Worked in the last Week						
	Labor Force Status	**0–19**	**20–29**	**30–39**	**40–49**	**50–59**	**60 or more**	**Total**
F	**Working Full-Time**	155	31	99	468	127	141	1021
P	**Working Part-Time**	171	54	60	32	5	9	331
	Total	326	85	159	500	132	150	1352

55. Working full-time

56. Working part-time and 0–19 hours

57. Working full-time and 40–49 hours

58. Working part-time or working less than 30 hours

EMPIRICAL PROBABILITY

USING HISTORY TO DETERMINE PROBABILITIES.

55. $P(\text{WORKING FULL TIME}) = \dfrac{n(\text{WORKING FULL TIME})}{n(S)}$

$= \dfrac{1021}{1352} = \boxed{.7552}$

56. $P(P \cap 0\text{-}19) = \dfrac{n(P \cap 0\text{-}19)}{n(S)} = \dfrac{171}{1352} = \boxed{.1265}$

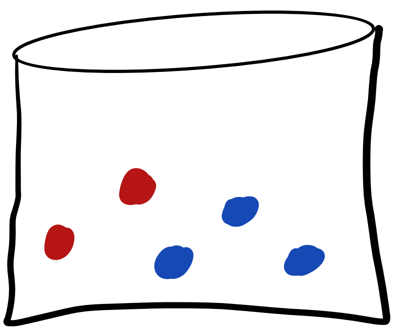
57. $P(F \cap 40\text{-}49) = \dfrac{468}{1352} = \boxed{.3462}$ OR $\boxed{34.62\%}$

58. $P(P \cup 0\text{-}29) = \dfrac{n(P \cup 0\text{-}29)}{n(S)} = \dfrac{155+31+171+54+60+32+5+9}{1352}$

$= \dfrac{517}{1352} = .3824$

ODDS

ODDS VS. PROBABILITY



EXPERIMENT: SELECT 1 MARBLE.

EVENT A: RED MARBLE.

$P(A) = \dfrac{n(A)}{n(S)} = \dfrac{2}{5} = .4 = 40\%$

ODDS OF A OCCURRING: $\dfrac{2}{3} \rightsquigarrow 2:3$

$P(A) = \dfrac{n(A)}{n(S)}$	ODDS OF A OCCURRING $= \dfrac{n(A)}{n(A')} \rightsquigarrow n(A):n(A')$ ASSUMING $n(A') \neq 0$

NOTE: ODDS OF A OCCURRING $= \dfrac{n(A)}{n(A')} = \dfrac{\frac{n(A)}{n(S)}}{\frac{n(A')}{n(S)}} = \dfrac{P(A)}{P(A')} \sim P(A):P(A')$

Example 6 Suppose the weather forecaster says that the probability of rain tomorrow is $1/3$. Find the odds in favor of rain tomorrow.

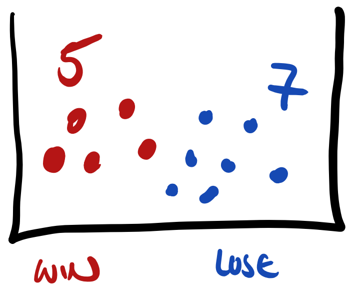
A = RAIN TOMORROW

ODDS OF A OCCURRING : $\dfrac{P(A)}{P(A')} = \dfrac{1/3}{1-1/3} = \dfrac{1/3}{2/3} \cdot \dfrac{3}{3} = \dfrac{1}{2}$

$\boxed{1:2}$

ex. SUPPOSE ODDS IN FAVOR OF A PARTICULAR HORSE WINNING THE KENTUCKY DERBY ARE 5 TO 7 (5 : 7).

FIND PROBABILITY.



$P(\text{WIN}) = \dfrac{5}{5+7} = \dfrac{5}{12} = .4167$

PROB → ODDS: ODDS IN FAVOR OF A $= \dfrac{P(A)}{P(A')} \sim P(A):P(A')$

ODDS → PROB : ODDS IN FAVOR OF A $= m:n$

$P(A) = \dfrac{m}{m+n}$

Example 9 **Social Science** Let A represent that the driver in a fatal crash was age 24 or younger and let B represent that the driver had a blood alcohol content (BAC) of .08% or higher. From data on fatal crashes in the year 2010 from the U.S. National Highway Traffic Safety Administration, we have the following probabilities.

$$P(A) = .2077, \quad P(B) = .2180, \quad P(A \cap B) = .0536.$$

(a) Find the probability that the driver was 25 or older and had a BAC level below .08%.

	≤ 24 A	> 24 A'	TOTALS
B	$.0536$	$.2180 - .0536 = .1644$	$.2180$
			+
B'	$.2077 - .0536 = .1541$	$.7820 - .1541 = .6279$	$.7820 = 1 - .2180$
TOTAL	$.2077$	+ $.7923$	$= 1$

$P(A') = 1 - P(A)$
$= 1 - .2077 = .7923$