§7.3 Introduction to Probability

Developed by Gamblers

Def: An **Experiment** is any activity with an observable result.

ex. Flipping a coin · result: H or T

A **Trial** is a single repetition of an Experiment

The results of each trial are called **outcomes**.

The set of all possible outcomes is the **Sample Space**

(Sample space is Universal set)

ex. Experiment: roll a 6-sided die once.

Sample space $S = \{1, 2, 3, 4, 5, 6\}$

ex. Experiment: flip a coin twice

Sample space $S = \{HH, HT, TH, TT\}$

visualize

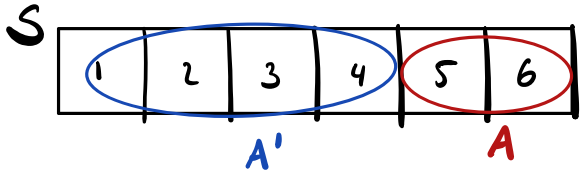
2nd Coin \ 1st Coin	H	T
H	HH	TH
T	HT	TT

Def: An **Event** is any subset of the Sample Space.

ex. EXPERIMENT: ROLL A 6-SIDED DIE ONCE.

SAMPLE SPACE $S = \{1, 2, 3, 4, 5, 6\}$

EVENT: A = ROLL A NUMBER > 4 $= \{5, 6\}$



COMPLEMENTARY EVENTS (EVENTS ARE SUBSETS OF SAMPLE SPACE)

A' = ROLL A NUMBER ≤ 4 $= \{1, 2, 3, 4\}$

GIVEN 2 EVENTS $A, B \subseteq S$

$A \cap B$: THE SET OF ALL OUTCOMES SUCH THAT BOTH EVENTS A & B OCCUR

$A \cup B$: THE SET OF ALL OUTCOMES SUCH THAT AT LEAST ONE OF THE EVENTS A OR B OCCURS.

ex. EXPERIMENT: ROLL A 6-SIDED DIE ONCE.

SAMPLE SPACE $S = \{1, 2, 3, 4, 5, 6\}$

EVENT: A = ROLL A NUMBER > 4 $= \{5, 6\}$

B = ROLL AN ODD NUMBER $= \{1, 3, 5\}$

NEW EVENTS

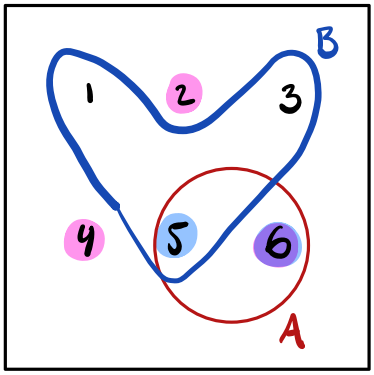
↳

$A \cap B$: ROLL ODD # > 4 $= \{5\}$

$A \cup B$: ROLL # > 4 OR ODD $= \{1, 3, 5, 6\}$

$A' \cap B$: ROLL # ≤ 4 AND ODD $= \{1, 3\}$

$A \cup B'$: ROLL # > 4 OR EVEN $= \{2, 4, 5, 6\}$



DEF: TWO EVENTS A & B ARE **MUTUALLY EXCLUSIVE** IF

$A \cap B = \emptyset$

(RECALL: TWO SETS A & B ARE DISJOINT IF $A \cap B = \emptyset$.)

ex. PREV. EXAMPLE: $A \cap B$ = ROLL ODD # > 4 = $\{5\} \neq \emptyset$ NOT EMPTY

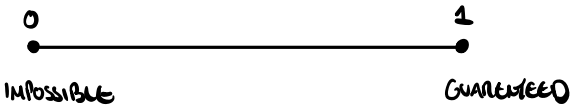
$\therefore$ A & B ARE NOT MUTUALLY EXCLUSIVE.

ex. GIVEN ANY EVENT $A \subseteq S$, WE KNOW $A \cap A' = \emptyset$

$\therefore$ A & A' ARE MUTUALLY EXCLUSIVE.

Def: TO EVERY EVERY EVENT A, WE ASSIGN A NUMBER $P(A)$ CALLED THE PROBABILITY OF A.

$$0 \leq P(A) \leq 1$$



$P(A)$ IS A MEASURE OF HOW LIKELY IT IS FOR A TO OCCUR.

ex. EXPERIMENT: FLIP A COIN TWICE. (EQUALLY LIKELY)

SAMPLE SPACE: $S = \{HH, HT, TH, TT\}$

WHAT IS THE PROBABILITY OF E = FLIPPING AT LEAST 1 HEAD?

Basic Probability Principle

Let S be a sample space of equally likely outcomes, and let event E be a subset of S. Then the **probability that event E occurs** is

$$P(E) = \frac{n(E)}{n(S)}.$$

($n(E)$: # WAYS E CAN HAPPEN; $n(S)$: # POSSIBLE OUTCOMES)

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{4} = .75 \quad 75\%$$

ex. Suppose we have 2 six-sided dice

1st die: P I Z Z A S

2nd die: P A Z A Z Z

Experiment: Roll both dice.

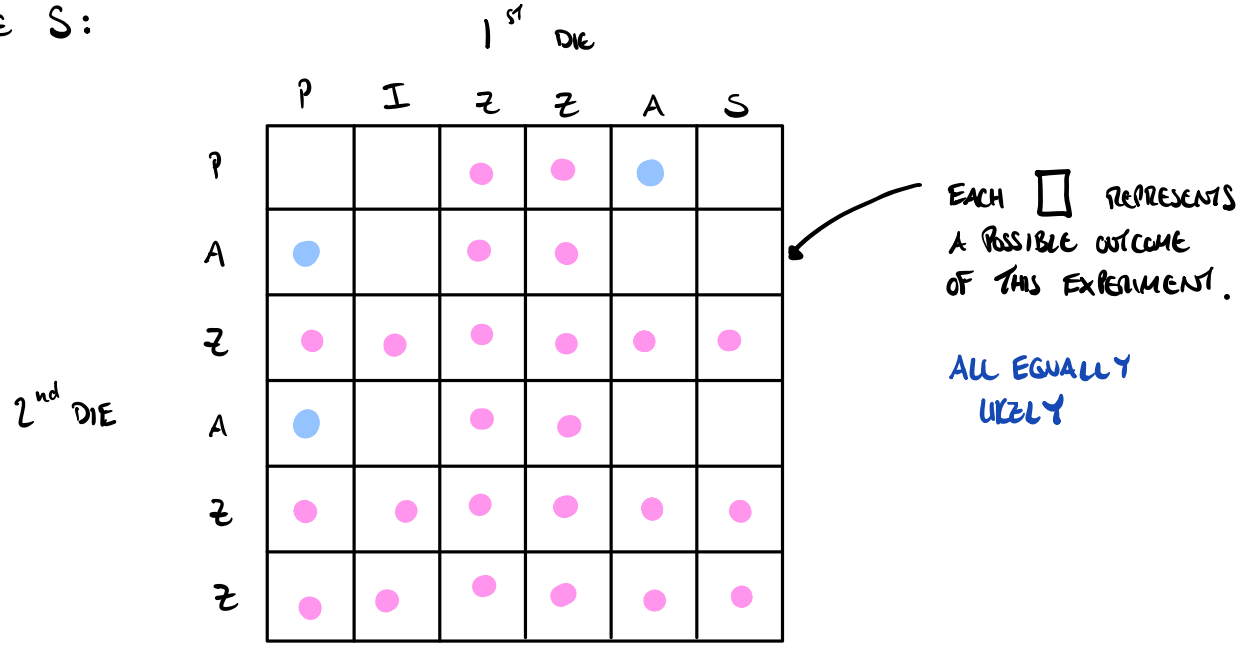
Event A: Roll at least one Z

Find P(A).

Step 1: Understand the Sample Space

S = { PP, PA, AP, SZ, ... ? }

Visualize S:



n(S) = # possible outcomes of experiment = 36

n(A) = # ways to roll at least one Z = 24

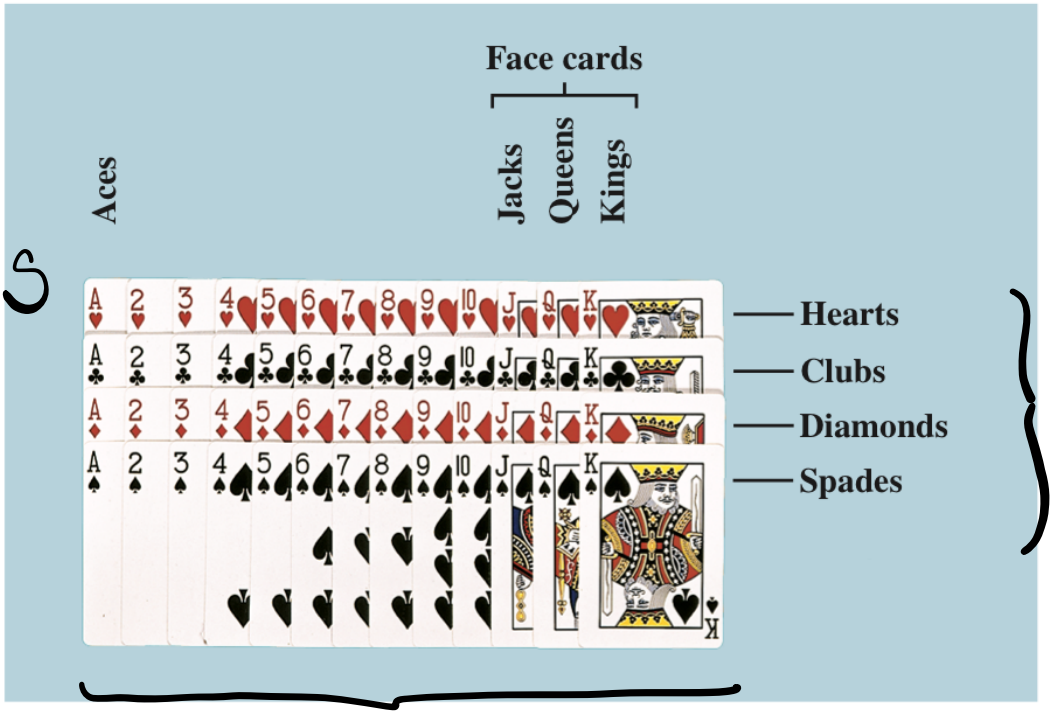
$$P(A) = \frac{n(A)}{n(S)} = \frac{24}{36} = \frac{2}{3} = .6667$$

↑ Between 0 & 1.

Event: B = Roll 1 P and 1 A.

$n(B) = 3$. $\Rightarrow P(B) = \frac{n(B)}{n(S)} = \frac{3}{36} = \frac{1}{12} = .0833$

ARE A & B MUTUALLY EXCLUSIVE? YES, $A \cap B = \emptyset$.



ROUND TO 4 DECIMAL PLACES

EX. EXPERIMENT: 1 CARD IS DRAWN FROM A STANDARD 52-CARD DECK.

LET A: DRAW A KING
B: DRAW A HEART

(a) FIND $P(A)$ & $P(B)$.

(a) $P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13} = .0769$

$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4} = .25$

(b) FIND $P(A \cap B)$ & $P(A \cup B)$.

(b) $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{52} = \boxed{.0192}$

$P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{n(A) + n(B) - n(A \cap B)}{n(S)}$

ADDITION RULE

$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$

$= P(A) + P(B) - P(A \cap B)$

$= .0769 + .25 - .0192 = .3077$

3. An experiment consists of rolling two fair 6-sided dice.
   - The first die has its faces labeled P, U, Z, Z, L, E.
   - The second die has its faces labeled S, I, Z, Z, L, E.

   (a) (6 points) What is the probabiliy that doubles are rolled (i.e. two of the same letter)?

   (b) (6 points) What is the probability that at least one Z is rolled?

4. An experiment consists of rolling 2 fair dice (all sides equally likely to be rolled). The first die has 4 sides labeled 1, 1, 2, 3. The second die has 6 sides labeled 1, 2, 2, 3, 3, 3.
   (a) What is the probablity that both dice show even numbers?
   (b) What is the probability that at least one die shows an odd number?
   (c) What is the probability that the sum the numbers rolled is odd?
   (d) What is the probabiity that at least one die is odd and the sum of the numbers rolled is even?

§7.2 REVIEW

3. Suppose A and B are two subsets of a universal set U such that $n(A) = 4$ and $n(B) = 6$.
   (a) If $n(U) = 12$, what are the possible values of $n(A \cap B)$ and $n(A \cup B)$?
   (b) If $n(U) = 8$, what are the possible values of $n(A \cap B)$ and $n(A \cup B)$?