§7.1 Sets

Def: SET

WELL-DEFINED COLLECTION OF OBJECTS (CALLED ELEMENTS) SUCH THAT IT IS ALWAYS POSSIBLE TO DETERMINE IF A GIVEN OBJECT IS INCLUDED IN THE COLLECTION OR NOT.

ex. DAYS OF THE WEEK

NOT. FAMOUS ACTORS   TOM HANKS (YES)
PETE HOLMES (MAYBE)

PARADOX: There is a village with one barber. The barber cuts the hair of everyone who does not cut their own hair, and only cuts the hair of people who do not cut their own hair.

Consider the set of people in this village who cut their own hair.
Is the barber in this set?

IF HE IS THEN HE ISN'T
IF HE ISN'T THEN HE IS $\Longrightarrow\!\!\!\times\!\!\!\Longleftarrow$ CONTRADICTION

NOT A SET.

ex. THE SET OF ALL SETS THAT DO NOT CONTAIN THEMSELVES.

DOES THIS SET CONTAIN ITSELF?

NOTE: WE WILL FOCUS ON SETS OF NUMBERS.

$\mathbb{N}$ THE SET OF NATURAL NUMBERS "COUNTING NUMBERS $\{1, 2, 3, \dots\}$

ELEMENTS OF A SET ARE LISTED INSIDE CURLY BRACKETS

$\mathbb{Z}$ THE SET OF INTEGERS $\{\dots, -2, -1, 0, 1, 2, 3, \dots\}$

ex. $A = \{2, 4, 6, 8\}$ $S = \{a, b, c\}$

SETS ARE OFTEN GIVEN NAMES, CAPITAL LETTERS.

A SET IS DETERMINED BY ITS ELEMENTS (CONTENTS) NOT BY THE ORDER OF ITS ELEMENTS.

$\{2,3,4\}$ & $\{3,4,2\}$ ARE THE SAME SET!

$\in$ LOWER CASE EPSILON (GREEK "e") : ELEMENT

$2 \in \{2,3,4\}$ 2 IS AN ELEMENT OF THE SET...
2 BELONGS TO THE SET ...

$5 \notin \{2,3,4\}$ 5 IS NOT AN ELEMENT OF THE SET.

EMPTY SET $\{\}$ THE SET THAT CONTAINS NO ELEMENTS. $\emptyset$

$0 \neq \emptyset \neq \{\emptyset\} = \{\{\}\}$

\# SET (EMPTY) SET WITH 1 ELEMENT, THE EMPTY SET.

ex. THE SET OF ALL HUMANS OVER 11 ft TALL.

Def: GIVEN A SET A, LET $n(A)$ = # ELEMENTS IN A.

e.g. $A = \{a, b, c\}$. THEN $n(A) = 3$.

e.g. $n(\emptyset) = 0$

$n(\{\emptyset\}) = 1$

Sometimes we describe a set by a common property of its elements rather than by a list of its elements. This common property can be expressed with **set-builder notation;** for example,

$$\{x \mid x \text{ has property } P\}$$ CURLY BRACKETS (SET)

(read, "the set of all elements x such that x has property P") represents the set of all elements x having some property P.

Example 1 List the elements belonging to each of the given sets.

(a) $\{x \mid x \text{ is a natural number less than } 5\}$

Solution The natural numbers less than 5 make up the set $\{1, 2, 3, 4\}$.

(b) $\{x \mid x \text{ is a state that borders Florida}\}$

Solution The states that border Florida make up the set {Alabama, Georgia}. ✓2

(c) $B = \{ \frac{p}{q} \mid p$ IS AN ODD INTEGER & q IS AN EVEN INTEGER $\}$

$\frac{3}{4} \in B$ ✓ $\frac{5}{8} \in B$ ✓

$\frac{5}{7} \notin B$ 7 IS NOT EVEN. DOG $\notin B$

SUBSETS: GIVEN 2 SETS A, B

A IS A SUBSET OF B $(A \subseteq B)$

IF EVERY ELEMENT OF A IS AN ELEMENT OF B.

e.g. $A = \{2, 4, 6\}$

$B = \{0, 2, 3, 4, 6, 9, 10\}$

THEN $A \subseteq B$, AND $B \nsubseteq A$

NOT EVERY ELEMENT OF B IS AN ELEMENT OF A.

$0 \in B$

$0 \notin A$

THEREFORE $B \nsubseteq A$.

B IS NOT A SUBSET OF A

Def: Two sets A, B are equal if $A \subseteq B$ and $B \subseteq A$.

In this case we write $A = B$.

(#'s: $a = b$ if $a \leq b$ and $b \leq a$.)

$A \subset B$

Def: A is a proper subset of B if $A \subseteq B$ and $A \neq B$.

e.g. $A = \{2, 4, 6\}$

$B = \{0, 2, 3, 4, 6, 9, 10\}$

$\underline{A \subseteq B}$, $B \not\subseteq A$, so $\underline{A \neq B}$.

$A \subset B$

(#'s $a \leq b$, $a \neq b$ then $a < b$.)

Note: Given any set A,
- $\emptyset \subseteq A$ empty set
- $A \subseteq A$ set itself.

Question: List all possible subsets of $A = \{a\}$

Set with just 1 element
" singleton "

$\emptyset$, A (2 subsets of a set with 1 element)

Question: List all possible subsets of $A = \{a, b\}$

0 elements $\to$ $\emptyset$,

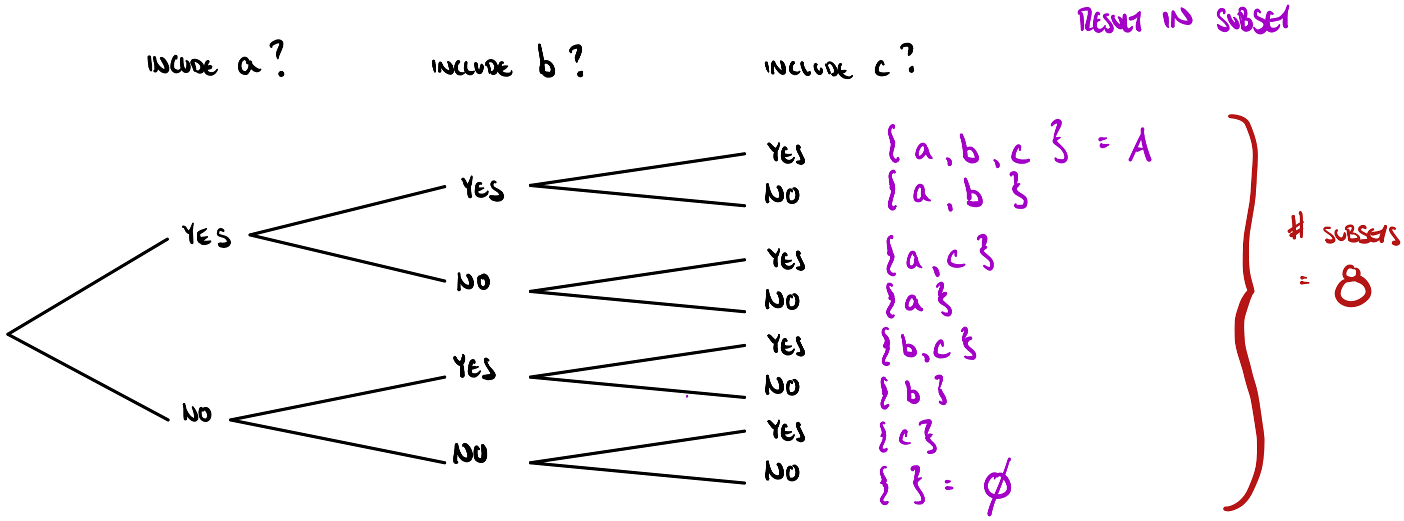
1 element $\to$ $\{a\}$, $\{b\}$

2 elements $\to$ $\{a, b\}$

(4 subsets of a set with 2 elements)

Question: How many possible subsets are there of $A = \{a, b, c\}$

Create a subset through a sequence of Yes/No Questions:



Binary Tree

$\boxed{\text{Theorem: A set with } k \text{ elements has } 2^k \text{ subsets}}$

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ex. An Uber driver must plan their schedule for the week by choosing which day(s) they will work (or not work).

e.g. Schedule 1 : Mon, Wed, Fri

Schedule 2 : Sun

⋮

How many possible schedules exist?

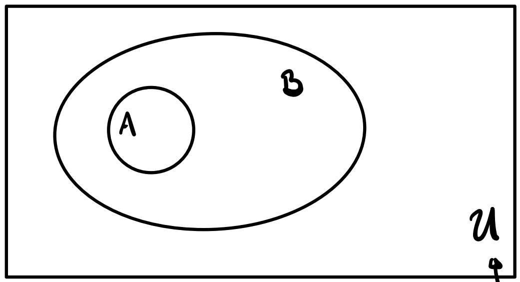
ex. What if Uber driver must work at least one day?

ex. What if Uber driver must work at least one day AND must not work at least one day?

Venn Diagrams:

Sets are represented by boxes & circles.

$A \subseteq B$



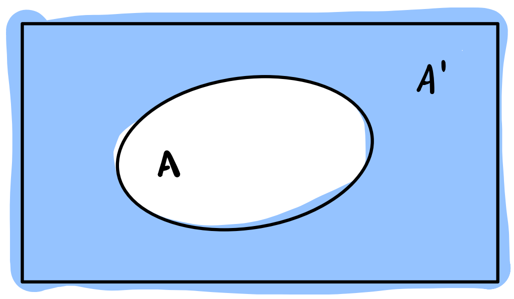
UNIVERSAL SET

Contains everything under consideration.
(context)

Set Operations

: Creating new sets from old sets

Complement A' ← Apostrophe



Def: The complement of a set A (denoted A') is the set of all elements of the universal set that are not elements of A.

e.g. $A = \{a, e, i, o, u\}$ "Vowels"

$A' = \{b, c, d, f, \ldots, y, z\}$, ~~my phone~~, ~~the 1 train~~, ~~traffic lights~~

Consonants

From context: universal set $\mathcal{U}$ = alphabet.

ex. $\mathcal{U} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

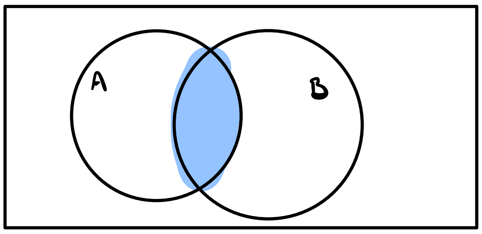
$A = \{1, 2, 3, 4\}$

$A' = \{0, 5, 6, 7, 8, 9\}$

Note: $(A')' = A$

INTERSECTION

THE INTERSECTION OF 2 SETS A AND B $(A \cap B)$ IS THE SET OF ALL ELEMENTS IN UNIVERSAL SET THAT BELONG TO BOTH A & B.

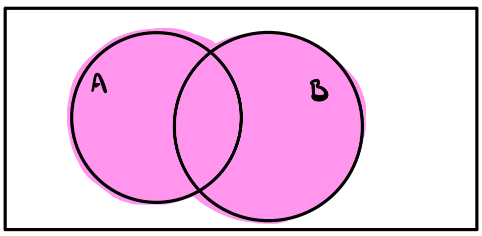


$A \cap B$ (OVERLAP)

$\cup$ FOR UNION

UNION

THE UNION OF 2 SETS A AND B $(A \cup B)$ IS THE SET OF ALL ELEMENTS THAT BELONG TO A OR B (OR BOTH A AND B)



$A \cup B$

ex. TALENT AGENT HAS MANY CLIENTS $(\mathcal{U})$.
SOME LIVE IN NY (N)
SOME OWN A CAR (C)
SOME ARE CURRENTLY EMPLOYED (E).

IN WORDS...

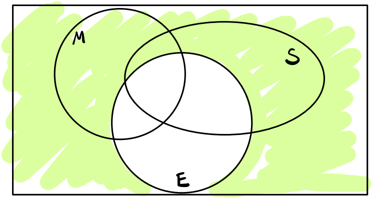
(a) $N \cap E$

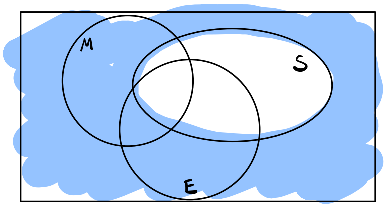
(b) $N' \cap C$

(c) $N' \cup C'$

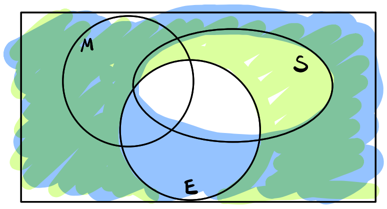
(d) $E' \cup C'$

SHADE IN REGIONS OF VENN DIAGRAM?



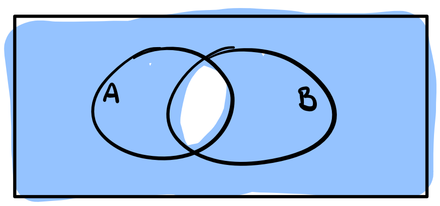


$E' \quad \cup \quad S'$

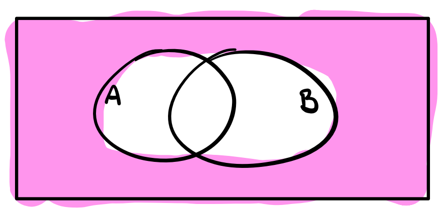
$=$ 

$(E \cap S)'$

IN GENERAL: **DEMORGAN'S LAW**



$A' \cup B' = (A \cap B)'$



$A' \cap B' = (A \cup B)'$

36. Is it possible for two nonempty sets to have the same intersection and union? If so, give an example.

Let $U = \{a, b, c, d, e, f, 1, 2, 3, 4, 5, 6\}$, $X = \{a, b, c, 1, 2, 3\}$, $Y = \{b, d, f, 1, 3, 5\}$, *and* $Z = \{b, d, 2, 3, 5\}$.

List the members of each of the given sets, using set braces. (See Examples 6–8.)

37. $X \cap Y$

38. $X \cup Y$

39. X'

40. Y'

41. $X' \cap Y'$

42. $X' \cap Z$

43. $X \cup (Y \cap Z)$

44. $Y \cap (X \cup Z)$

43. $Y \cap Z = \{b, d, 3, 5\}$

$X \cup (Y \cap Z)$:

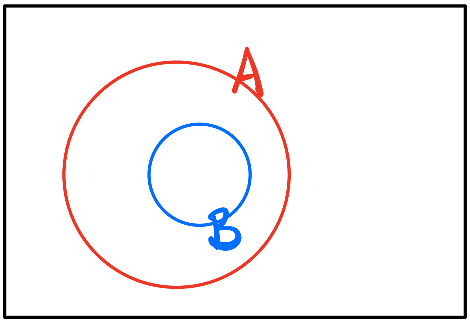
$\{a,b,c,1,2,3\} \cup \{b,d,3,5\}$

$= \{a,b,c,1,2,3,\cancel{b},d,\cancel{3},5\}$

$= \{a,b,c,d,1,2,3,5\}$

Extreme Intersections

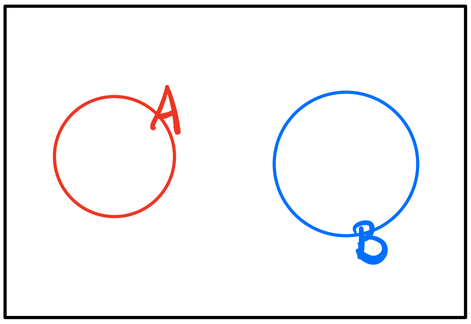
$A \cap B = B$



B is subset of A

$(B \subseteq A)$

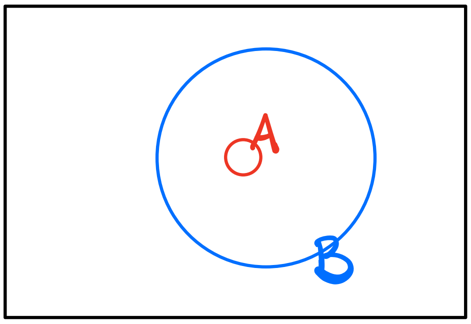
$A \cap B = \emptyset$



A & B are disjoint

$(A \cap B = \emptyset)$

$A \cap B = A$



A is a subset of B

$(A \subseteq B)$