

5.2 Future Value of an Annuity

Geometric Sequences

Example 1. Find the next 3 terms of each of the following sequences.

(a) 1, 6, 11, 16, 21, ...

(b) 3, 5, 8, 12, 17, ...

(c) 1, 4, 9, 16, 25, ...

(d) 3, 6, 12, 24, 48, ...

Definition 1. For any nonzero numbers a and $r \neq 1$, the sequence

$$a, ar, ar^2, ar^3, ar^4, \dots$$

is called a **geometric sequence**. The sum of the first n terms of a geometric sequence is

$$S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}.$$

Sum of Terms

If a geometric sequence has first term a and common ratio r , then the sum S_n of the first n terms is given by

$$S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1.$$

Example 2. Find the sum

$$500 + 500(1.005) + 500(1.005)^2 + 500(1.005)^3 + \dots + 500(1.005)^{359}.$$

Ordinary Annuities

Example 3. In order to save money for retirement, Zoe decides to deposit \$500 at the end of every month for the next 30 years into an account that earns 6% annual interest compounded monthly. What is her account balance 30 years later? How much money did she deposit in total?

Definition 2. A sequence of equal payments made at equal periods of time is called an **annuity**. If the payments are made at the end of the time period, and if the frequency of payments is the same as the frequency of compounding, the annuity is called an **ordinary annuity**. The time between payments is the **payment period**, and the time from the beginning of the first payment period to the end of the last payment period is called the **term** of the annuity. The **future value of the annuity**, the final sum on deposit, is defined as the sum of the compound amounts of all the payments, compounded to the end of the term.

Future Value of an Ordinary Annuity

$$S = R \left[\frac{(1 + i)^n - 1}{i} \right] \quad \text{or} \quad S = Rs_{\overline{n}|i}$$

where

S is the future value;

R is the periodic payment;

i is the interest rate per period;

n is the number of periods.

Example 4. In order to pay for their daughter to go to college, two new parents make periodic payments of R dollars at the end of every month for 18 years into an account that earns 4.8% annual interest compounded monthly. Find R such that the parents have \$50,000 saved for their daughter's college education.